Threat, Commitment, and Brinkmanship in Adversarial Bargaining*

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November 17, 2022

Job Market Paper

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Abstract

We study commitment strategies in adversarial bargaining—situations in which the proposer demands payment from the responder under the threat of a conflict. The bargaining environment is risky; a welfare-destroying conflict might randomly start during the negotiation, but it also can be induced by the proposer. If the conflict starts, the responder's loss depends on the scale of the conflict the proposer chooses, and it is maximized for a larger scale than the one that maximizes the proposer's benefit. The proposer has pre-commitment power to a scale given by audience costs. We describe under which conditions the proposer commits to start the conflict to threaten the responder and when she relies on the risky environment as a threat. We also show that having high pre-commitment power can be detrimental for the proposer because of a commitment trap. The proposer prefers a risky environment if she has high pre-commitment and a safer one in the opposite case.

Keywords: Bargaining, Commitment, Brinkmanship, Threat.

JEL Codes: C72, C78, D74.

*We thank Juan Ortner, Bart Lipman, Dilip Mookherjee, Kfir Eliaz, Anna Pechenkina, Chiara Margaria, and participants at the Public Choice Society Conference (2022), Stony Brook International Conference on Game Theory (2022) and USU brownbag workshop for helpful comments and suggestions.

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1 Introduction

Negotiators reduce the scope of their own authority, and confront the management with the threat of a strike that the union itself cannot avert, even though it was the union's own action that eliminated its power to prevent the strike.

An Essay on Bargaining (1956)
Thomas C. Schelling

In adversarial bargaining—situations in which an agent demands payment from another under the threat of a conflict—the ability to commit to a threat plays an important role. In an international conflict between two countries in which one of them demands territory from the other under the threat of starting a war, the demanding country's bargaining power increases if that country can credibly commit to starting the war if the demand is not satisfied.

But the demanding party will not necessarily be committed to the war. If the threatened country decides not to satisfy the demand, will the demanding country indeed start the war? A war is a welfare-destroying activity, and even if the demanding country benefits from starting it, if the losses of the other country are higher than the demanding country's benefits, there are incentives to continue negotiating instead of actually starting a war. The same problem is found in many different situations. A prosecutor offers a plea deal to a defendant under the threat of subjecting him to a trial if the deal is not accepted, or a labor union threatens to start a strike if the employer does not raise their salaries.

Furthermore, if the demanding party starts the war, is the promised intensity enough to persuade the threatened country to accept a demand? The demanding party can choose the scale of conflict as a threat to the other party—for example, the mobilization of troops or charges before plea bargaining. If the scale is subject to a scaling-down cost, how can the demanding party use those scaling-down costs to optimally threaten the other party? In the international conflict case, scaling-down cost takes the form of audience costs, which are the "domestic political costs a leader may pay for escalating an international dispute, or for making implicit or explicit threats, and then backing down"[1] A similar form of reputational costs applies to plea bargaining and union-employer negotiations.

The objective of this paper is to study the demanding party's commitment strategies. We provide conditions under which the demanding party can credibly commit to starting the conflict. We also show that higher scaling-down costs might be detrimental in some cases

[1] James Fearon, "Credibility is not everything but it's not nothing either." September 2013. URL: https://themonkeycage.org/2013/09/credibility-is-not-everything-either-but-its-not-nothing-either/

for the demanding party. The role of scaling-down costs has mainly been recognized as a positive feature for the party who has it. We show that this is not always the case, since it can generate a commitment trap for the demanding party.

We consider a multiperiod bargaining model between a proposer (she) and a responder (he) under the risk of a conflict. The negotiation continues as long as the conflict has not started. We model the conflict as a reduced-form function that assigns payoffs depending on the conflict scale, which the proposer chooses. At the beginning of each period, the proposer chooses a promised scale of the conflict to threaten the responder. The proposer can later modify the promised scale if the conflict actually starts, as described in the following paragraph. After choosing the promised scale, the proposer makes an offer that consists of a payment from the responder to the proposer. If the offer is accepted, the game ends. If the offer is rejected, the proposer decides whether to start the conflict or continue negotiating. We consider a risky negotiation environment: Even if the proposer chooses not to start the conflict, it exogenously starts with some probability at the end of each period.

The proposer chooses any promised scale at each period, but she faces a scaling-down cost of choosing a lower scale than in the previous period. The scaling-down cost is increasing in the magnitude of the reduction of the scale from one period to another. If the conflict starts, the proposer has the last opportunity to revisit the scale before the payoffs are realized, subject to the same scaling-down costs. The scale is bounded above by an upper bound.

The responder gets a loss in the conflict, which is increasing on the scale. The proposer receives a payoff that decreases on the scale, which is positive (a benefit) if the scale is low and negative (a loss) if the scale is large. This implies that the scale that maximizes the responder's loss in the conflict is larger than the one that maximizes the proposer's benefit. It means that the proposer has incentives to threaten the responder on a large scale during the negotiation but to scale it down if the conflict actually starts.

Whether the proposer can credibly commit to starting the conflict depends on the scaling-down costs and the probability that it exogenously starts. If the probability that the conflict exogenously starts is high, the proposer relies on it and decides not to start the conflict if the responder rejects the offer. The proposer chooses a large scale during the negotiation as a way of tying her hands and inducing herself to choose a large revisited scale if the conflict starts. The proposer puts herself in a position in which she gets a significant loss if the conflict starts. This implies that she will not start the conflict if the responder rejects an offer because it generates a loss for her. However, given that the exogenous probability of the conflict is high, the responder accepts a high offer because his expected loss in the conflict is also high.

In contrast, if the probability that the conflict exogenously starts is low, relying on it is not optimal. Suppose the proposer chooses not to start the conflict after a rejection. In that case, the responder is willing to accept only a low offer because his expected loss of waiting for the conflict to exogenously start is low. Therefore, the proposer chooses a scale such that if her offer is rejected, she would weakly prefer to start the conflict than continue negotiating. The scale needs to be low because the proposer's payoff has to be positive (and higher or equal to the continuation value of continuing the negotiation) to prefer to start the conflict. Therefore, by choosing a low conflict scale, she credibly commits to starting the conflict if an offer is rejected. This gives her the bargaining power to make a high offer and be accepted by the responder, since the responder gets a loss if the conflict starts.

If the probability that the conflict exogenously starts takes intermediate values, whether the proposer commits to start the conflict depends on the scaling-down costs. If those costs are high, the proposer does not start the conflict and chooses a large scale to maximize the responder's loss in case the conflict starts. On the contrary, if the scaling-down costs are low, the proposer credibly commits to starting the conflict because even by promising a large conflict scale, she will decrease it to a low one if the conflict starts.

The result formalizes the definitions Thomas Schelling provides for the types of threats. He defines a *probabilistic threat* as one that "the threatener may carry out, or maybe not, if the second party fails to comply... The motive may be that what is threatened is of enormous size."[2] The case in which the proposer relies on the probability of the exogenous shock corresponds to a probabilistic threat. Furthermore, the equilibrium corresponds to a *brinkmanship equilibrium* if the scaling-down costs are high, since the *enormous size* puts the proposer in a position of having a loss. Schelling refers to brinkmanship as "exploiting the danger that somebody may inadvertently go over the brink, dragging the other with him."[3] We refer to the case in which the proposer credibly commits to start the conflict as a *deterministic threat*.

If the probability that the conflict exogenously starts is low or high, the optimal decision whether to start the conflict does not change depending on the value of the scaling-down costs. In that case, the proposer is better off with high scaling-down costs. However, for intermediate values of the probability that the conflict exogenously starts, whether the proposer decides to start the conflict depends on the scaling-down costs—so it might not be that higher scaling-down costs are better for the proposer.

Suppose the probability that the conflict exogenously starts takes intermediate values. The offer the responder is willing to accept is higher under a deterministic threat with a low scale than under a probabilistic threat with a large scale. This is because the probability that

[2]See (25).
[3]See (24).

the conflict exogenously starts is not high enough. Then the responder is willing to accept a higher offer if he knows the conflict will surely start if he rejects the offer. Therefore, the proposer prefers to choose a low scale to commit to starting the conflict if the responder rejects an offer. The scale needs to be low because it provides her a positive payoff of the conflict that incentivizes her to start it.

However, starting the conflict is not credible if the proposer has high scaling-down costs. In that case, the high scaling-down costs are detrimental to the proposer because she faces a commitment trap. She would like to commit to a low scale of the conflict because the offer the responder is willing to accept is higher if the proposer commits to start the conflict, but it is not credible.

If the responder rejects an offer, when deciding whether to start the conflict or continue negotiating the proposer will prefer to continue negotiating and increase the scale of the conflict. Increasing the scale of the conflict to maximize the responder's loss is optimal. The offer the responder is willing to accept under a probabilistic threat is higher than the proposer's payoff of starting the conflict. This is because the responder's expected loss of waiting for the conflict to start is not low enough, given that the probability the conflict exogenously starts is not low enough. The proposer can increase the scale to the one that generates the highest loss to the responder precisely because her scaling-down costs are high. When, instead, the proposer has low scaling-down costs, she cannot credibly commit to a scale that maximizes the responder's loss, which renders it credible that she will start the conflict following a rejection of the offer. Starting the conflict is not sequentially rational for the proposer if she has high scaling-down costs.

Our model delivers the following comparative statics. If the proposer has high scaling-down costs, she is better off negotiating in a risky environment than in a safer one. She can induce brinkmanship by putting herself in a position in which, if there is conflict, she gets a loss—but that also maximizes the loss for the responder, which increases her bargaining power. If the proposer has low scaling-down costs, she is better off negotiating in a safer environment than in a risky one. In that case, the proposer cannot commit to a scale that maximizes the responder's loss. So, she prefers a safer environment to credibly commit to starting the conflict. The responder is better off with the lowest risk that induces a probabilistic threat.

Lastly, we extend our model to one in which the proposer is privately informed about her scaling-down costs. We introduce a rational full-commitment proposer, who rationally chooses the scale at the first negotiation period and does not modify it. We show that the commitment trap still exists for high values of scaling-down costs if the prior belief regarding the proposer's being the full-commitment type is not high enough. For lower scaling-down costs, the commitment trap is lessened because the proposer mimics the full-commitment type, which increases her payoff respect to the case without private information.

Related Literature: This paper is related to the pre-commitment literature, which starts with (22) and (23), who viewed the bargaining process as an attempt by players to commit themselves to a position and credibly convince others that conceding is not possible. (7) formalizes Schelling's ideas with a two-period bilateral bargaining model, in which players attempt to tie their hand in the first period, and in the second period they decide whether to back down from their commitment. (19) and (20) formalize the scaling-down cost and make it proportional to the size of the concession.

(8) extends Muthoo's model to include arbitrary scaling-down costs to concede. (9) extends (8) to characterize sequential concessions in an infinite-horizon model. (17) study an infinity horizon bargaining model in which the proposer can commit in each round, but commitment is costly and lasts only one round.[4]

(5) and (bas) apply the pre-commitment model to a political competition in which public opinion plays a role in the concession decision. (15) and (27) apply the model to international conflict negotiations.

There are two main differences between previous literature and our paper: First, in previous models, players first commit to a demand, and then there is a negotiation that can last either infinitely or for one period. Although we allow concessions during the negotiation, in this paper we include the conflict as the last stage, which allows us to study the commitment decision regarding the threat of a potentially harmful action for everyone if a demand is not satisfied. It allows us to analyze the credibility of the threat—that is, what the scale is and whether the proposer will rely on the risky environment or commit to starting the conflict.

The second difference is that we focus on adversarial bargaining. Instead of the mutual benefits of an agreement, we consider that one player demands a payoff from another. This difference changes the incentives for agreement; in bargaining with mutual benefits, all players prefer no delay, but in adversarial bargaining, the player facing a loss prefers to delay the agreement.

Among papers that focus on starting a conflict rather than pre-commitment to a position, (26) present a bilateral conflict model in which one party can start a conflict that ends the game with a negative payoff for both players. They show that if the proposer can divide the conflict into small "attacks" that do not end the game, the proposer gains bargaining power and can extract all of the responder surplus as a sequence of transfers on the equilibrium path, and conflict is avoided. In the present paper, we focus on the commitment strategies

[4] Papers that study commitment previous to the negotiation, but in which the commitment can fail, are by (11); (16); (6); (12); (18).

that derive from the interaction between pre-commitment power (given by the scaling-down costs) and the scale of the threat. Also, we focus on a more general and realistic setting in which a player can reap benefits from the conflict, although the total welfare of the economy is reduced.

Our analysis of the inclusion of a full-commitment type is related to the reputational literature, in which there is no pre-commitment stage. Instead, one of the players can be, with a small probability, a behavioral type that never changes her demand. The type is private information; therefore, a rational player can pretend to be the behavioral type. (2) present the canonical model, and several extensions have been made, such as by (14); (3); (28); (4); (21); and (10).[5]

(21) posits that the behavioral type is a rational player with full commitment. The behavioral type optimally chooses the initial demand and cannot change it in later rounds. We consider a similar full-commitment type, who behaves rationally but cannot modify her scale of the threat.

Outline: The paper is organized as follows. Section 2 introduces the model, Section 3 analyzes the equilibrium, Section 4 introduces private information and the presence of a full-commitment type, and Section 5 concludes. Appendix A contains extensions, and Appendix B has all the proofs.

2 Model

There are two players: a proposer (she) and a responder (he). They play an adversarial bargaining game in which the proposer demands a payment from the responder. The game has two stages: the *negotiation stage*, in which players try to get to an agreement, and the *resolution stage*, in which the conflict starts.

The game starts with the multiperiod negotiation stage, in which the proposer and responder negotiate an agreement. At the end of each period, $t \in \mathbb{N} \equiv \{1, 2, 3, ...\}$ in the negotiation stage; if the proposer and responder do not reach an agreement, the proposer decides whether to start the resolution stage. If the proposer chooses not to start the resolution stage, a shock exogenously starts it with probability p, and a new negotiation period starts with probability $(1 - p)$.

Negotiation stage: In each period $t \in \mathbb{N} \equiv \{1, 2, 3, ...\}$, the proposer chooses an *intended scale of the conflict* $x_I^t \in [0, \bar{x}]$, which affects the payoffs at the resolution stage. After she chooses

[5] In (10), the rational type can make an ultimatum to start a conflict (e.g., a war), but in their framework, the outcome of the conflict depends solely on the players' type and not on the threats.

the *intended scale*, the proposer offers a deal y to the responder. A deal is a payment from the responder to the proposer. The game ends if the responder accepts the deal. In that case, the proposer gets a payoff of y and the responder a loss of y. If the offer is rejected, a new negotiation period starts with probability $(1-p)$ if the proposer decides not to start the resolution stage. For each period $t \geq 2$, the proposer has a scaling-down cost $kc(x_I^{t-1}, x_I^t)$ of decreasing the intended scale from x_I^{t-1} to x_I^t, in which $c(x_I^{t-1}, x_I^t)$ is an increasing and convex function in the difference $x_I^{t-1} - x_I^t$, with $c(x_I^{t-1}, x_I^t) = 0$ if $x_I^{t-1} \leq x_I^t$, and $c(x_I^{t-1}, x_I^t) > 0$ if $x_I^{t-1} > x_I^t$. We call *pre-commitment power* to the parameter $k \geq 0$.

Resolution stage: This stage contains the conflict, which is a reduced-form function that assigns benefit and loss depending on the proposer's *realized scale of the conflict*. Before the conflict begins, the proposer has the chance to change her intended scale x_I^t to a realized scale x_F at a cost $kc(x_I^t, x_F)$, as described above. The responder's loss function in the conflict is $u_R(x)$, and the proposer's benefit function is $u_P(x)$. Benefit and loss are not symmetric, since the conflict is a welfare-destroying activity. We discuss this assumption in Section 2.1. Both functions are continuous and concave over $[0, \bar{x}]$ with a unique maximizer. The unique maximizers $x^P = \arg\max_{x\in[0,\bar{x}]} u_P(x)$ and $x^R = \arg\max_{x\in[0,\bar{x}]} u_R(x)$ are related in the following way:[6]

$$0 < x^P < x^R < \bar{x}.$$

The maximum of each function, $u_P(x^P)$ and $u_R(x^R)$, are strictly higher than zero (but remember that $u_R(x^R)$ is a loss). We restrict attention to $u_P(x) < u_R(x)$ for $x \in (0, \bar{x}]$, as shown in Figure 1. That is, there is welfare destruction in the resolution stage. For example, in an international conflict, the invading country gets new territory but loses troops and faces international sanctions; the prosecutor can damage her reputation for losing at the trial, and the union leader might face retaliation in the future.[7]

[6] In the main part of the paper we focus on the case in which $x^R < \bar{x}$, and relegate to Appendix A.1 the case $x^R = \bar{x}$.

[7] We relegate to Appendix A.1 the case $u_R(x^P) < u_P(x^P) \leq u_R(x^R)$. In that case, there is no welfare destruction for all scales but only for some values. For example, in the international conflict case, the invading country is demanding a territory with a resource that is strategic for them but is less valued by the country that initially has it. In the plea bargaining case, the prosecutor might have high-powered private benefits (for example, media exposure) of "winning" at trial.

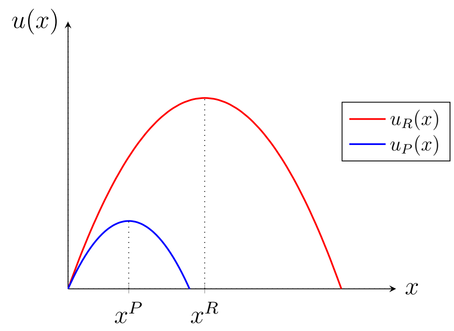


Figure 1: Proposer's benefit and responder's loss in the conflict.

Both players discount the future with a discount factor $\delta \in [\underline{\delta}, 1)$.

Parametric assumptions: We assume $u_P(x^R) < 0$. That is, the scale that maximizes the responder's loss also generates a loss to the proposer. We also assume that the discount factor is bounded below by $\underline{\delta} = \frac{u_P(x^P)}{u_R(x^P)}$. This implies, in principle, that the proposer has incentives to continue negotiating after a rejection.

2.1 Discussion of assumptions

1. The benefit and loss function: The model captures the conflict between two parties. It highlights how the proposer uses pre-commitment power to threaten the responder and how it affects outcomes. Considering that $u_R(x) > u_P(x)$ for all x means that the proposer prefers to induce the responder to accept the offer instead of directly starting the resolution stage. Further, considering that $x^R < x^P$ means that the proposer has incentives to choose a large intended scale in the negotiation but a more conservative scale at the resolution stage.

If $x^R = x^P$, there is no trade-off in choosing the scale, and if $u_R(x) < u_P(x)$ for all x, the proposer prefers to start the resolution stage directly. Finally, if $x^R > x^P$, under slight modifications, the same intuition applies in the opposite direction.

There is welfare destruction at the resolution stage, because the proposer cannot capture all of the responder's losses. For example, after invading a country and successfully annexing new territories, the demanding country faces international sanctions and a loss of credibility. A proposer can *win* at trial but also *lose*, which affects her reputation and career options.

2. Scaling-down costs: Scaling-down costs are the to audience or reputation costs of reducing the intended scale. (13) introduces audience costs in international conflict, and we extend the intuition to any situation in which a leader faces electoral costs, as in the case of a union leader. Reputation costs are similar. For example, reducing the charges before trial might be interpreted as bad prosecutorial practice and hurt the prosecutor's future career.

3. Probability that a shock exogenously starts the resolution stage: The resolution stage might exogenously begin in some cases. For example, a general on the border might trigger an attack without authorization, which starts a war; and the judge could reject postponing a trial (with probability p), which ends the plea bargaining. Not considering the external shock is equivalent to setting $p = 0$, a particular case of our model.

3 Analysis

In this section, we show that the proposer's credibility regarding starting the resolution stage is determined by the value of p and k. If p is low enough or takes intermediate values, but k is also low enough, the proposer can promise an appropriate scale that makes credible the threat of starting the resolution stage if the responder rejects an offer. In the opposite case, the proposer does not commit to starting it and relies on the probability that the exogenous shock will start it. We also show that having high pre-commitment power might be detrimental for the proposer for intermediate values of p, since the proposer faces a high commitment trap.

To formalize Schelling's definitions regarding the nature of a threat, we define the threat as probabilistic and deterministic:

- *Probabilistic Threat:* The proposer does not start the resolution stage after a rejection of the offer. Instead, she chooses a scale and the threat is given by the chance that the resolution stage exogenously starts.
- *Deterministic Threat:* The proposer starts the resolution stage after a rejection of the offer, and it is credibly communicated to the responder.

The equilibrium concept is subgame Nash perfect equilibrium (SNPE). We say that an equilibrium is a *brinkmanship equilibrium* if it features a probabilistic threat, and the proposer chooses an intended scale such that the realized scale generates the largest loss to the responder and also a loss to the proposer.

We show the results and describe the equilibrium in detail following backward induction. We start with the resolution stage, then discuss the choice of the intended scale.

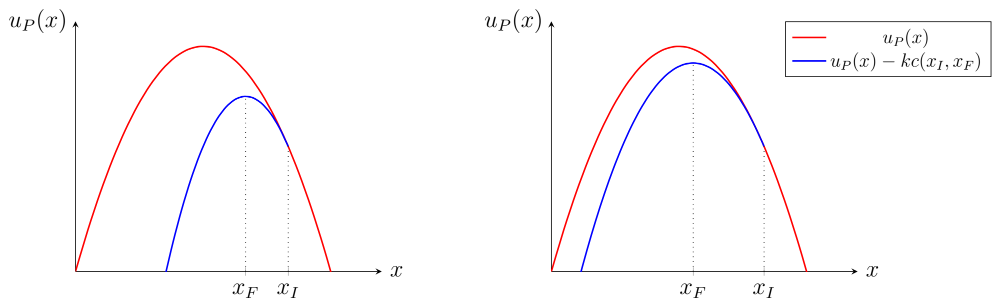
3.1 Realized scale at the resolution stage

Suppose the proposer chose x_I as the intended scale in the last period before the resolution stage started. We define $x(x_I, k)$ as the realized scale x_F given x_I and k. That is:

$$x(x_I, k) \equiv \arg\max_{x\in[0,\bar{x}]} u_P(x) - kc(x, x_I).$$

Lemma 1 *$x(x_I, k)$ is strictly increasing in x_I for $x_I \in [0, \bar{x}]$, and strictly increasing in k for $x_I \in [x^P, \bar{x}]$.*

Figure 2 shows the function $u_P(x) - kc(x, x_I)$. Note that if $x_I > x^P$, then $x(x_I, k) < x_I$ for any x_I and $k > 0$. If $x_I = x^P$, then $x(x_I, k) = x_I$ for any $k \geq 0$. And if $x_I < x^P$, then $x(x_I, k) = x^P$ for any $k \geq 0$, as there is no cost of increasing the scale.



(a) High k

(b) Low k

Figure 2: Proposer's payoff at resolution stage after choosing x_I.

The realized scale $x(x_I, k)$ is increasing in both parameters, since if the intended scale is higher for any k, the realized scale will be higher. Also, for any intended scale, the realized scale is increasing in the cost k; for a larger k, any decrease in the scale is more expensive. The proposer optimally never increases the scale in the resolution stage if $x_I \geq x^P$, since x^P is the optimal scale without considering the scaling-down cost. Increasing the scale has no cost for any $x_I < x^P$; therefore, the realized scale will be x^P.

Figure 3 shows the realized scale $x_F = x(x_I, k)$ as a function of x_I for several k. In the figure, to get a realized scale $x_F = x^R$, the optimal promised threat is x^R if $k \to \infty$, x^1 if $k = k_1$, and $\bar{x}$ if $k = \bar{k}$; it is not feasible for $k = k_2$ and $k = 0$.

We define $X_F(k)$ as the set of feasible realized scales at the resolution stage for pre-commitment power k:

$$X_F(k) \equiv [x^P, x(\bar{x}, k)].$$

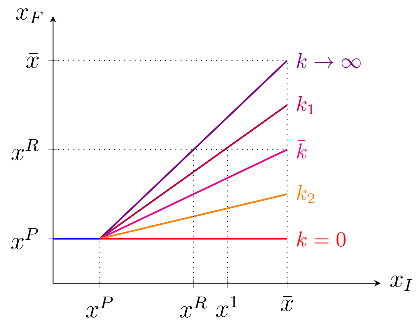


Figure 3: Realized scale depending on the intended scale.

3.2 Decisions at the negotiation stage

We first analyze the highest offer the responder is willing to accept, which depends on the realized scale and credibility of the threat. Then we analyze under which conditions the proposer decides whether to start the resolution stage and the optimal x_I she chooses.

3.2.1 Responder's willingness to accept the offer

The responder benefits from delaying an agreement. He is getting a loss; therefore, not reaching an immediate deal is good for him. However, in equilibrium, the responder accepts the offer in the first period.

If the proposer chooses x_I at t, for any k, the responder's continuation value if he rejects the offer at period t is

$$V_R(x_I, k) = \begin{cases} p u_R(x(x_I, k)) + (1-p)\delta V_R^{t+1}(x_I^{t+1}, k) & \text{if proposer does not start the resolution stage,} \\ u_R(x(x_I, k)) & \text{if proposer starts the resolution stage,} \end{cases}$$

in which $V_R^{t+1}(x_I^{t+1}, k)$ is the responder's continuation value if there is a new negotiation period $t+1$. If the proposer chooses a stationary x_I at t—that is, she does not change it during the negotiation—the above expression becomes:

$$V_R(x_I, k) = \begin{cases} \tilde{p} u_R(x(x_I, k)) & \text{if proposer does not start the resolution stage,} \\ u_R(x(x_I, k)) & \text{if proposer starts the resolution stage.} \end{cases}$$

In that case, $\tilde{p}$ represents the *composed probability of the resolution stage* in a stationary equilibrium, which takes high values if p is high. It is defined as

$$\tilde{p} = \frac{p}{1-(1-p)\delta} \; .$$

Then, the optimal offer the proposer makes is equal to the continuation value, and it depends on whether the proposer starts the resolution stage.

3.2.2 Proposer's decision whether to start the resolution stage

The proposer decides to start the resolution stage following a rejection of the offer if two conditions are satisfied:

1. The proposer's payoff at the resolution stage is higher than waiting for one more period under the same scale x_I.

$$u_P(x(x_I, k)) - kc(x(x_I, k), x_I) \geq \delta \tilde{p} u_R(x(x_I, k)). \tag{1}$$

2. The proposer's payoff at the resolution stage is higher than waiting one more period with a different scale x'_I.

$$u_P(x(x_I, k)) - kc(x(x_I, k), x_I) \geq \delta V_P(x_I^{t+1}, k), \tag{2}$$

   in which $V_P(x_I^{t+1}, k)$ represents the proposer's continuation value of choosing x_I^{t+1} in case she decides not to start the resolution stage and it is not started by the shock.

Note that during the negotiation, the proposer does not reduce the scale because it is costly. The proposer is better off directly choosing a lower x_I than choosing a higher one and scaling it down in subsequent periods. However, the proposer might have incentives to increase the scale in subsequent periods because it has no cost.

Conditions (1) and (2) are satisfied depending on the values of $\tilde{p}$ and k, and the optimal selection of x_I.

3.2.3 Probabilistic and deterministic threat

Whether the equilibrium is a probabilistic or deterministic threat depends on the pre-commitment power and $\tilde{p}$. We define two cutoffs, $\tilde{p}_H$ and $\tilde{p}_L$, for the value $\tilde{p}$:

$$\tilde{p}_H = \frac{u_P(x^P)}{\delta u_R(x^P)} \quad \text{and} \quad \tilde{p}_L = \frac{u_P(x^P)}{\delta u_R(x^R)} \ .$$

We also define k^* as the k such that

$$u_P(x(\bar{x}, k)) - kc(x(\bar{x}, k), \bar{x}) = \delta \tilde{p} u_R(x(\bar{x}, k)) \ .$$

Proposition 1 *The equilibrium depends on $\tilde{p}$ and k as follows:*

- *The equilibrium features a probabilistic threat for any k if $\tilde{p} > \tilde{p}_H$, and for $k > k^*$ if $\tilde{p} \in [\tilde{p}_L, \tilde{p}_H]$. The proposer chooses an optimal intended threat $x_I(k, \tilde{p})$ and offers $y = \tilde{p}u_R(x_F(k, \tilde{p}))$, which is accepted by the proposer at $t = 1$.*

- *The equilibrium features a deterministic threat for any k if $\tilde{p} < \tilde{p}_L$, and for $k \leq k^*$ if $\tilde{p} \in [\tilde{p}_L, \tilde{p}_H]$. The proposer chooses an optimal intended threat $x_I(k, \tilde{p})$ and offers $y = u_R(x_F(k, \tilde{p}))$, which is accepted by the proposer at $t = 1$.*

In what follows, we describe the optimal $x_I(k, \tilde{p})$ for each case of the value $\tilde{p}$ and the equilibrium details.

If $\tilde{\mathbf{p}}$ is high. If $\tilde{p}$ is high enough ($\tilde{p} > \tilde{p}_H$), there is no x_I and its respective $x_F \in [x^P, \bar{x}]$ given by $x(x_I, k)$ that satisfy condition (1). Therefore, it is not sequentially rational to start the resolution stage for any intended scale x_I and pre-commitment power k.

Intuitively, the proposer's highest payoff of starting the resolution stage is given by choosing $x_I = x^P$, the realized scale of which is $x_F = x^P$. In that case, at the moment of deciding whether to start the resolution stage, the proposer is better off not starting it, because the responder's loss of rejecting any offer (assuming the proposer never starts the resolution stage) is $\tilde{p}u_R(x^P)$, and:

$$u_P(x^P) < pu_P(x^P) + (1 - p)\delta\tilde{p}u_R(x^P) \quad \Longleftrightarrow \quad u_P(x^P) < \delta\tilde{p}u_R(x^P),$$

which holds for $\tilde{p} > \tilde{p}_H$. Therefore, for any x_I and k, it is not sequentially rational for the proposer to start the resolution stage.

The value $\tilde{p}_H$ is the $\tilde{p}$ that renders the proposer indifferent between starting the resolution stage or waiting one more period if $x_I = x^P$. If $x_F > x^P$, then $u_P(x_F) < \delta\tilde{p}_H u_R(x_F)$. Therefore, for any k the proposer decides not to start the resolution stage. The optimal offer the proposer makes to the responder is $y = \tilde{p}u_R(x)$, which is accepted. The off-the-path-of-equilibrium strategy of not starting the resolution stage, but which exogenously starts with a high probability, supports the strategy of accepting the offer.

The proposer uses the probability that an exogenous shock starts the resolution stage to threaten the responder. The proposer maximizes her payoff by choosing at $t = 1$ a threat x_I that maximizes the offer $y = \tilde{p}u_R(x(x_I, k))$. The optimal $x_I(k)$ the proposer chooses at period $t = 1$ is

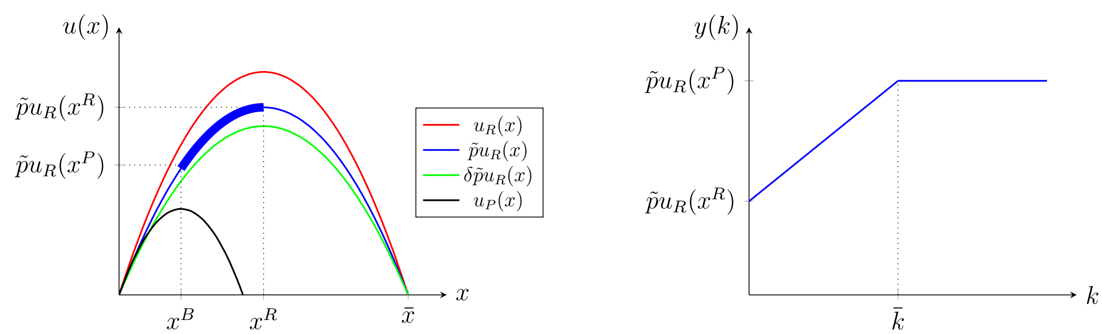
$$x_I(k) = \begin{cases} \bar{x} & \text{if } k \leq \bar{k} \\ x_I \text{ such that } x(x_I, k) = x^R & \text{if } k > \bar{k} \end{cases}$$

and the proposer does not change the intended scale at any period $t \geq 2$ in the negotiation stage.

For $\tilde{p} > \tilde{p}_H$ the proposer maximizes the offer the responder is willing to accept using a probabilistic threat. Furthermore, if $k > \bar{k}$ the equilibrium is brinkmanship, since the threat is probabilistic, and the realized scale if the resolution stage starts maximizes the responder's loss and gives a negative payoff to the proposer.

The proposer maximizes the offer the responder is willing to accept by choosing a x_I such that $x(x_I, k) = x^R$ as long as $k \geq \bar{k}$, because $x^R \in X_F(k)$. To achieve x^R as the realized scale, the proposer chooses $x_I > x^R$ such that at the resolution stage she scales it down to $x(x_I, k) = x^R$. The optimal x_I is increasing in k. The proposer offers $y = \tilde{p}u_R(x^R)$ to the responder. The responder accepts $\tilde{p}u_R(x^R)$ because it is equal to his continuation value evaluated at x^R.

If k is lower ($k < \bar{k}$), the scale that maximizes the responder's loss does not belong to the feasible set of realized scale: $x^R \notin X_F(k)$. The best the proposer can do is to choose $\bar{x}$ and offer $y = \tilde{p}u_R(x(\bar{x}, k))$ to the responder, and the responder accepts it because it is equal to his continuation loss evaluated at $x(\bar{x}, k)$. In this case, the proposer gets a larger payoff at the resolution stage compared with $k \geq \bar{k}$. Figure 4 Panel (a) shows the responder's induced continuation loss, which translates into the optimal offer in Figure 4 Panel (b).



(a) Responder's induced continuation loss

(b) Optimal offer

Figure 4: Payoff, loss, and offers if $\bar{p} > \bar{p}_H$. The set of optimal offers is highlighted in bold in Panel (a).

If $\tilde{\mathbf{p}}$ is low. If $\tilde{p} \leq \tilde{p}_L$, the proposer can choose an appropriate x_I to induce herself to start the resolution stage after a rejection of the offer for any k. For any k, the proposer can choose $x_I = x^P$ to satisfy conditions (1) and (2). If $x_I = x^P$, condition (1) is satisfied because

$$u_P(x^P) > pu_P(x^P) + (1-p)\delta\tilde{p}u_R(x^P) \quad \Longleftrightarrow \quad u_P(x^P) > \delta\tilde{p}u_R(x^P),$$

which is true for $\tilde{p} < \tilde{p}_H$. Condition (2) is satisfied because

$$u_P(x^P) > pu_P(x^P) + (1-p)\delta\tilde{p}u_R(x^R) \quad \Longleftrightarrow \quad u_P(x^P) > \delta\tilde{p}u_R(x^R),$$

which is true for $\tilde{p} < \tilde{p}_L$. In this case, the best deviation $x'_I > x_I$ the proposer can choose after a rejection if the resolution stage does not start is x_I such that $x(x_I, k) = x^R$ if $k \geq \bar{k}$, as shown above. Therefore, if $\tilde{p} \leq \tilde{p}_L$, for any k, there is x_I that induces the proposer to start the resolution stage.

Indeed, in this case, the proposer prefers to start the resolution stage. She chooses the optimal x_I that satisfies conditions (1) and (2) because the offer the proposer can induce the responder to accept under a deterministic threat is higher than that under a probabilistic threat. The lowest offer the proposer can induce the responder to accept under a deterministic threat is $u_R(x^P)$, which is higher than the highest offer the proposer can induce the responder to accept under probabilistic threat $\tilde{p}u_R(x^R)$ because $\tilde{p}$ is small.[8]

Therefore, the proposer chooses an appropriate x_I so that she starts the resolution stage if the offer is rejected. In this case, the proposer targets a more conservative realized scale than in the case $\tilde{p} > \tilde{p}_H$ because a necessary condition to start the resolution stage is to get a positive payoff for it. The optimal x_I the proposer chooses at the beginning of $t = 1$ is

$$x_I(k) = \begin{cases} \bar{x} & \text{if } k \leq \underline{k} \\ x_I \text{ such that } u_P(x(x_I, k)) - kc(x(x_I, k), x_I) = \delta\tilde{p}u_R(x(\bar{x}, k)) & \text{if } k \in [\underline{k}, \bar{k}] \\ x_I \text{ such that } u_P(x(x_I, k)) - kc(x(x_I, k), x_I) = \delta\tilde{p}u_R(x^R) & \text{if } k > \bar{k}, \end{cases}$$

and the proposer does not change the intended scale at any period $t \geq 2$ in the negotiation stage.

If the proposer's pre-commitment power is infinitely high ($k \to \infty$), we denote by x^B the optimal scale that satisfies conditions (1) and (2). The value of x^B is the x_I that satisfies:

$$u_P(x_I) = \delta\tilde{p}u_R(x^R).$$

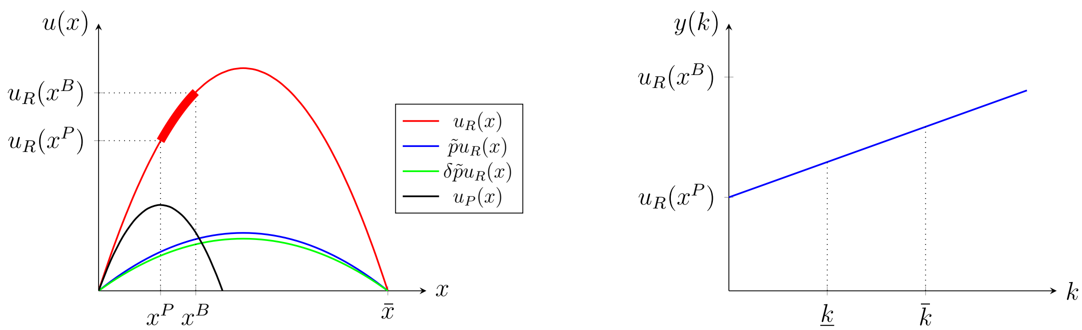
A lower scale induces the responder to accept a lower offer, and a higher one does not satisfy (2). Suppose the proposer chooses a higher scale $x'_I > x^B$ such that $u_P(x'_I) > \delta\tilde{p}u_R(x'_I)$ but $u_P(x'_I) < \delta\tilde{p}u_R(x^R)$. In that case, the proposer satisfies condition (1) because the proposer is better off starting the resolution stage than not starting it, considering that x'_I is stationary. However, x'_I is not stationary, since the proposer is better off by choosing not

[8] In this case $u_R(x^P) > \delta^{-1}u_P(x^P)$ by the parametric assumption, and $\delta^{-1}u_P(x^P) > \tilde{p}u_R(x^R)$ because $\tilde{p} < \tilde{p}_L$.

to start the resolution stage after a rejection of the offer and increase the scale to $x_I = x^R$ because the expected payoff of increasing it is $pu_P(x_I') + (1-p)\delta\tilde{p}u_P(x^R)$, which is higher than $u_P(x_I')$ if $u_P(x_I') < \delta\tilde{p}u_R(x^R)$.

The proposer chooses the closest value to x^B that satisfies (1) and (2). That is, for any k, the realized scale is $x(x_I, k) \in (x^P, x^B)$. If k is low enough, the best the proposer can do is to choose $\bar{x}$, which satisfies (1) because the scaling-down costs are low, and (2) because there is no option to increase the scale. Denote by $\underline{k}$ the k that satisfies (1) with equality by choosing $x_I = \bar{x}$.[9]

Figure 5 Panel (a) shows the value x^B. The proposer's realized scale off the path of equilibrium is given by $x(x_I(k), k) = x(k)$, which is increasing, and $x(0) = x^P$ and $\lim_{k\to\infty} x_F(k) = x^B$.



(a) Responder's induced continuation loss

(b) Optimal offer

Figure 5: Continuation loss for $\tilde{p} < \tilde{p}_L$ and optimal intended scale. The set of optimal offers is highlighted in black.

If $\tilde{\mathbf{p}}$ takes an intermediate value. In the previous cases, whether the optimal threat is probabilistic or deterministic only depends on the value $\tilde{p}$. However, for $\tilde{p} \in [\tilde{p}_L, \tilde{p}_H]$ it also depends on the pre-commitment power k. We show that in this case, if k is high, the threat is probabilistic and if k is low the threat is deterministic.

First, we show that if k is low enough, the only option for the proposer is to start the resolution stage after a rejection.

Lemma 2 *For $\tilde{p} < \tilde{p}_H$, if $k \leq \underline{k}$, the proposer starts the resolution stage following a rejection.*

As defined in the previous case, $\tilde{k}$ is the k that satisfies condition (1) with equality by choosing $x_I = \bar{x}$. This implies that if $k \leq \tilde{k}$, for any x_I condition (1) and (2) are satisfied.

[9] $\underline{k}$ is the k such that $u_P(x(\bar{x}, k)) - kc(x(\bar{x}, k), \bar{x}) = \delta\tilde{p}u_R(x(\bar{x}, k))$.

Condition (2) is satisfied because after choosing x_I as the intended scale, there is no deviation $x'_I > x_I$ the proposer chooses after a rejection that changes the threat from deterministic to probabilistic. This implies that the proposer is better off by choosing the optimal x_I that maximizes the offer the responder is willing to accept under a deterministic threat, which is $x_I = \bar{x}$. Intuitively, the pre-commitment power is very low, and therefore even by choosing a large intended scale, the proposer will scale it down to a realized scale that generates a positive payoff of starting the resolution stage, which implies that is sequentially rational to start it for the proposer.

Second, if k is high enough, the opposite case is true: The proposer does not start the resolution stage following a rejection. We define $\tilde{k}$ as the k such that $u_P(x^P) = \delta\tilde{p}u_R(x(\bar{x}, k)$, that is, the k for which condition (2) is satisfied by choosing $x_I = x^P$ if $\tilde{p} \geq \tilde{p}_L$.

Lemma 3 *For $\tilde{p} > \tilde{p}_L$, if $k \geq \tilde{k}$, the proposer does not start the resolution stage following a rejection.*

If $k > \tilde{k}$, condition (2) is not satisfied for any x_I. Suppose the proposer chooses $x_I = x^P$, which satisfies condition (1) and also provides the higher payoff to the proposer at the resolution stage $u_P(x^P)$. Even by choosing x^P the proposer will prefer to choose not to start the resolution stage and, instead, will continue negotiating with a higher intended scale. Suppose $k \geq \tilde{k}$ is high enough that $x^R \in X_F(k)$—that is, the proposer can choose an x_I such that the realized scale is x^R. If the proposer decides to increase the scale to one that delivers x^R as the realized scale, her expected payoff is

$$pu_P(x^P) + (1-p)\delta\tilde{p}u_R(x^R),$$

which is larger than $u_P(x^P)$ for $\tilde{p} > \tilde{p}_L$. Therefore, even by choosing the highest x_I that maximizes the proposer's payoff at the resolution stage, condition (2) is not satisfied.

From Lemmas 2 and 3, if $k \in [\underline{k}, \tilde{k}]$, whether the threat is deterministic or probabilistic depends on the value x_I the proposer chooses. The value x_I that maximizes the offer the responder is willing to accept under a deterministic threat is $x_{DT}(k)$, which is the x_I that satisfies

$$u_P(x(x_I, k)) - kc(x(x_I, k), x_I) = \delta\tilde{p}u_R(x(\bar{x}, k)).$$

The proposer decides to induce herself to start the resolution stage if $k < k^*$, in which k^* is the pre-commitment power that renders the proposer indifferent between a probabilistic and deterministic threat:

$$u_R(x(x_{DT}(k), k)) = \tilde{p}u_R(x(\bar{x}, k)).$$

In period $t = 1$ the proposer chooses

$$x_I(k) = \begin{cases} \bar{x} & \text{if } k \leq \underline{k} \\ x_I \text{ such that } u_P(x(x_I, k)) - kc(x(x_I, k), x_I) = \delta \tilde{p} u_R(x(\bar{x}, k)) & \text{if } k \in [\underline{k}, k^*] \\ \bar{x} & \text{if } k \in [k^*, \bar{k}] \\ x_I \text{ such that } x(x_I, k) = x^R & \text{if } k > \bar{k} \end{cases}$$

and the proposer does not change the initial threat at any period $t \geq 2$ in the negotiation stage.[10]

The intuition of the described optimal intended scale can be divided in two. If $k \geq k^*$, the equilibrium threat is a probabilistic threat, and if $k < k^*$, the threat is deterministic. In each of the described equilibrium types, the proposer targets the intended scale that maximizes her payoff.

If $k \geq k^*$, the intented scale that maximizes the proposer's payoff is x^R or the closes possible that k allows. Following the same intuition as $\tilde{p} > \tilde{p}_H$, the proposer chooses x_I such that $x(x_I, k) = x^R$ if k is high enough, and $x = \bar{x}$ if $k \leq \bar{k}$. If $k < k^*$, the optimal intended scale is the highest x_F that satisfies conditions (1) and (2). If $k \in [\underline{k}, k^*]$, she can choose a $x < \bar{x}$ that satisfies but conditions, but if $k > \underline{k}$ the best that she can do it to choose $x_I = \bar{x}$.

3.3 High pre-commitment power can be detrimental

For $\tilde{p} \in [\tilde{p}_L, \tilde{p}_H]$, whether the offer the responder is willing to accept is higher under a deterministic or probabilistic threat depends on the value of $\tilde{p}$. Intuitively, if $\tilde{p}$ takes a high value, the offer the responder is willing to accept is higher under a probabilistic threat. If $\tilde{p}$ takes a low value, then the responder is willing to accept a higher offer under a deterministic threat. We define $\tilde{p}_M$ as the $\tilde{p}$ such that

$$u_R(x^P) = \tilde{p}_M u_R(x^R);$$

that is, if $\tilde{p} \leq \tilde{p}_M$, the proposer is better off inducing the responder to accept $u_R(x^P)$ than $\tilde{p} u_R(x^R)$. Note that $u_R(x^P)$ is the lowest offer the responder is willing to accept under a deterministic threat, and $\tilde{p} u_R(x^R)$ is the highest offer the responder is willing to accept under a probabilistic threat.

In that case, if $\tilde{p} \leq \tilde{p}_M$, the proposer would like to choose a x_I that induces a deterministic threat. However, as discussed in Lemma 3, if $k > \tilde{k}$, it is not possible to choose a x_I that

[10]Note that $\tilde{k} \in [k^*, \bar{k}]$.

satisfies condition (2).

Proposition 2 *If $\tilde{p} \in [\tilde{p}_L, \tilde{p}_M]$, the offer the proposer induces the responder to accept is higher for any $k < \tilde{k}$ than for any $k \geq \tilde{k}$.*

Proposition 2 shows that the proposer is better off with low commitment power k if the value of $\tilde{p}$ takes intermediate values. As Figure 6 shows, if $\tilde{p} \in [\tilde{p}_L, \tilde{p}_M]$, the proposer is better off under a deterministic threat. However, the deterministic threat is not feasible for $k > \tilde{k}$. The proposer with high pre-commitment power faces a *commitment trap*.

Definition: We define a *commitment trap* as a situation in which the proposer with high pre-commitment power would like to choose a low scale to induce a deterministic threat, but the high pre-commitment power makes it noncredible.

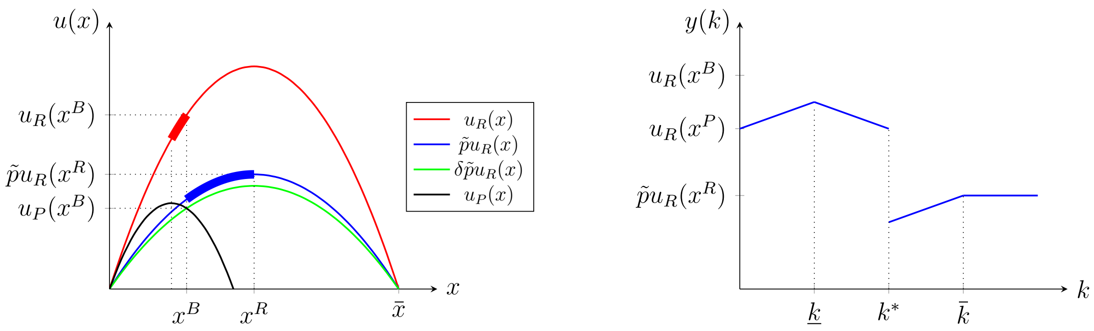
The proposer with high pre-commitment power would like to choose a low intended scale to induce herself to start the resolution stage following a rejection of the offer. However, it is not sequentially rational because it does not satisfy condition (2). Suppose the extreme case in which $k \to \infty$. Suppose further that the proposer chooses a $x_I = x^P$ that satisfies (1) because $u_P(x^P) > \delta\tilde{p}u_R(x^P)$ for $\tilde{p} < \tilde{p}_H$. The scale $x_I = x^P$ provides the highest payoff to the proposer at the resolution stage. Even for that x_I, if the responder rejects an offer and the proposer has to decide whether to start the resolution stage or continue negotiating, she would prefer to continue negotiating and to choose $x_I = x^R$ instead of starting the resolution stage because the expected payoff is higher:

$$pu_P(x^P) + (1-p)\delta\tilde{p}u_R(x^R) > u_P(x^P),$$

since $u_P(x^P) > \delta\tilde{p}u_R(x^R)$ because $\tilde{p} > \tilde{p}_L$.

Therefore, a deterministic threat is noncredible for the proposer if the pre-commitment is high, which generates the commitment trap.

If the pre-commitment is low, she would also like to choose x^R as the realized scale instead of starting the resolution stage, but she cannot do it because her pre-commitment power does not support x^R as a feasible scale. Therefore, the commitment trap does not affect the proposer if her pre-commitment is low. Paradoxically, a low pre-commitment power allows the proposer to "commit" to a low scale, making a deterministic threat credible.



(a) Responder's induced continuation loss

(b) Optimal offer

Figure 6: Payoff, loss, and offers if $\bar{p} \in [\bar{p}_L, \bar{p}_M]$. The set of optimal offers is highlighted in bold in Panel (a).

3.4 Riskier or safer environment?

The probability of exogenous shock p represents how risky the negotiation environment is—that is, how likely it is that the resolution stage will exogenously start and destroy welfare. We showed that the proposer takes advantage of a risky environment ($\tilde{p} > \tilde{p}_H$) by generating a brinkmanship equilibrium if k is high. If the environment is less risky ($\tilde{p} < \tilde{p}_L$), the responder is willing to accept $u_R(x)$, which is higher than $\tilde{p}u_R(x)$.

If $k < \underline{k}$, the proposer is better off in a safer environment (low p) than in a risky one (except if $\tilde{p} = 0$). And if $k > \bar{k}$, the proposer can take advantage of the risky environment, and therefore she is better off with high p. Figure 7 shows the highest offer the responder is willing to accept as a function of $\tilde{p}$.

If $k < \underline{k}$, the proposer's payoff is $u_R(x(\bar{x}, k))$ for any $\tilde{p} < \tilde{p}_H$. If $\tilde{p} \geq \tilde{p}_H$, her payoff is $\tilde{p}u_R(x(\bar{x}, k))$, which is lower than $u_R(x(\bar{x}, k))$ for $\tilde{p} < 1$. If $k > \bar{k}$, the proposer targets x^R as the realized scale for any $\tilde{p} \geq \tilde{p}_L$, and the highest x_I that satisfy conditions (1) and (2) if $\tilde{p} > \tilde{p}_L$. Therefore, if $\tilde{p} \geq \tilde{p}_L$ her payoff is $\tilde{p}u_R(x^R)$ and if $\tilde{p} < \tilde{p}_L$ the payoff is $u_R(x_F)$ in which $x_F < x^R$.

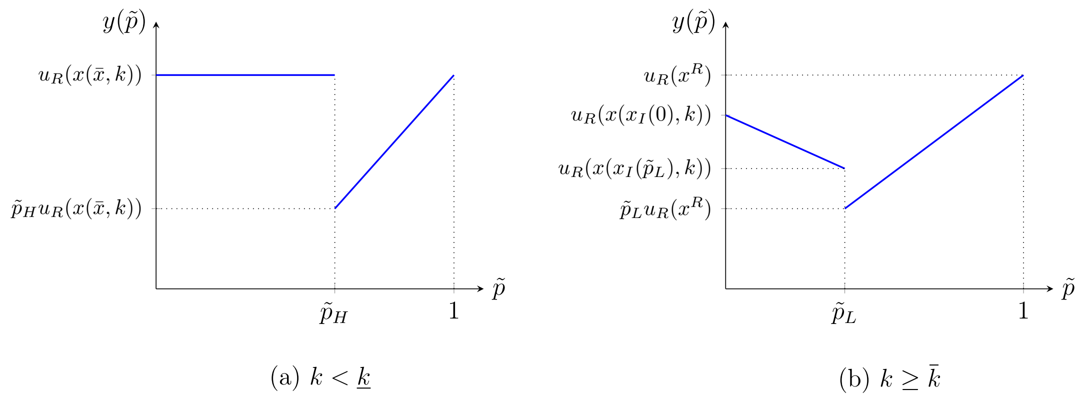


Figure 7: Highest offer the responder is willing to accept as a function of $\tilde{p}$ for $k < \underline{k}$ and $k > \bar{k}$.

3.5 Wrapping up

The equilibrium takes the form of a deterministic threat equilibrium if the risk that the conflict will exogenously start is low because relying on that risk to threaten the proposer is not effective. In that case, the proposer prefers to start the resolution stage. If it is high, the proposer can rely on it, since it generates enough loss for the responder to accept a high offer. In this case, if the pre-commitment power is high enough for the proposer to generate the highest loss for the responder, the equilibrium is brinkmanship; the proposer puts herself in a position in which if the resolution stage exogenously starts, she will incur a loss.

If the probability that a shock will start the resolution stage takes intermediate values, the proposer relies on the risky environment if the pre-commitment is high. In this case, there might exist a commitment trap; if the pre-commitment is high, the proposer would like to choose a more conservative scale and commit to starting the resolution stage, since it generates a higher payoff. However, this is not sequentially rational, since the proposer is better off not starting the resolution stage and increasing the threat if an offer is rejected.

Lastly, the proposer prefers to negotiate in a safer environment if she has low pre-commitment power. If she has high pre-commitment power, she prefers a risky environment to carry out the negotiation.

4 Private information

In this section, we answer the question of whether the commitment trap is robust to the presence of private information. Following the reputational bargaining literature, we introduce a full-commitment proposer type who behaves rationally but cannot change her intended scale. The two proposer types we consider are

- *Full-commitment type*: Does not change the intended scale $x_I = x_F$, and
- *Partial-commitment type*: The pre-commitment power is given by k.

Both players discount the future with the same discount factor $\delta \in [\underline{\delta}, 1)$. The proposer's type is the proposer's private information, and it is denoted by $\alpha \in \{L, H\}$. We denote the full-commitment type as $\alpha = H$ and the partial commitment type as $\alpha = L$. The responder's prior belief about the proposer's being the full-commitment type is $P(\alpha = H) = \theta \in [0, 1]$.

We show that the commitment trap still exists under private information if the prior belief regarding the proposer's being the full-commitment type is low, or if the prior belief takes intermediate values and the pre-commitment power is high. We further show that in general, private information benefits the proposer only for low values of pre-commitment power.

The equilibrium concept is perfect Bayesian equilibrium (PBE). Depending on the responder's beliefs specification, there are potentially many equilibria. To analyze under which condition the partial-commitment type benefits from the presence of the full-commitment type, we restrict attention to the equilibrium that maximizes the partial-commitment type's expected payoff.

We define $x_I^H(k, \tilde{p})$ as the optimal intended scale the full-commitment type chooses, and by $x_I^L(k, \tilde{p})$ and $x_F^L(x_I^L, k)$ the intended scale and the realized scale of the partial-commitment type, respectively. $x_I^*(k, \tilde{p})$ refers to the optimal intended scale the partial-commitment chooses if there is no private information. The value $x_I^*(k, \tilde{p})$ is described in Section 3. For notational simplicity, we write x_I^H, x_I^L, and x_I^*.

4.1 Commitment trap under private information

Under public information, the commitment trap exists for pre-commitment values $k > k^*$ if $\tilde{p} \in [\tilde{p}_L, \tilde{p}_M]$.

Suppose there is no private information and the proposer is the full-commitment type. In that case, the full-commitment type chooses the highest x that induces a deterministic threat as the intended scale. Note that for the full-commitment type to start the resolution stage, the only relevant condition is condition (1), since she cannot change her intended scale. We call x^{B2} the optimal intended scale for the full-commitment type. The value x^{B2} corresponds to the x that satisfies

$$u_P(x) = \delta \tilde{p} u_R(x).$$

Proposition 3 *In the partial-commitment type's payoff-maximizing equilibrium, for $\tilde{p} \in [\tilde{p}_L, \tilde{p}_M]$ and $k \geq k^*$, there exists $\bar{\theta}$ and $\underline{\theta}$ such that*

- *If $\theta < \underline{\theta}$, the equilibrium is separating with intended scale $x_I^L = x_I^*$, and x_I^H is the x such that $u_R(x) = \tilde{p}u_R(x(x_I^*, k))$.*
- *If $\theta > \bar{\theta}$, the equilibrium is pooling with intended scale $x_I^L = x_I^H = x^{B2}$.*
- *If $\theta \in [\underline{\theta}, \bar{\theta}]$, there exists a k^{Pool-H} such that the equilibrium is pooling if $k \in [k^*, k^{Pool-H}]$, and separating if $k > k^{Pool-H}$.*

Both proposer types make the same offer, and the responder accepts in the first period.

In this case, the full-commitment type starts the resolution stage following a rejection of the offer because, by definition, she cannot change her scale, and both x^{B2} and the x such that $u_R(x) = \tilde{p}u_R(x(x_I^*, k))$ satisfy condition (1).

In both cases—pooling and separating equilibrium—the full-commitment type starts the resolution stage following a rejection of the offer. By definition, she cannot change her scale, and both x^{B2} and the x such that $u_R(x) = \tilde{p}u_R(x(x_I^*, k))$ satisfy condition (1). However, if the equilibrium is pooling, the partial-commitment proposer does not start the resolution stage following a rejection. Instead, she increases the intended scale to continue negotiating. This implies that she reveals her type if there is a new negotiation period. The responder's posterior belief would be $\theta' = 0$, which starts a subgame as in the public information case. Therefore, the partial-commitment type's payoff-maximizing equilibrium is pooling if

$$\theta u_R(x^{B2}) + (1-\theta)\Big[pu_R(x(x^{B2}, k)) + (1-p)\delta\tilde{p}u_R(x(x_I^*, k))\Big] \geq \tilde{p}u_R(x(x_I^*, k)).$$

Call $\theta^*(k)$ the prior belief that satisfies the above expression with equality, which depends on the value k. The above equation can be written as

$$\theta \geq \theta^*(k) \equiv \frac{\tilde{p}u_R(x(x_I^*, k)) - \Big[pu_R(x(x^{B2}, k)) + (1-p)\delta\tilde{p}u_R(x(x_I^*, k))\Big]}{u_R(x^{B2}) - \Big[pu_R(x(x^{B2}, k)) + (1-p)\delta\tilde{p}u_R(x(x_I^*, k))\Big]};$$

that is, the equilibrium is pooling if $\theta \geq \theta^*(k)$. The expression $\theta^*(k)$ is increasing in k and is bounded below by $\underline{\theta} = \theta^*(k^*) > 0$ and above by $\bar{\theta} = \lim_{k\to\infty}\theta^*(k^*) < 1$. Figure 8 represents the function $\theta^*(k)$.

Define the value k^{Pool-H} as the inverse of $\theta^*(k)$, and therefore, for any $\theta \in [\underline{\theta}, \bar{\theta}]$, the equilibrium is pooling if $k < k^{Pool-H}$ and separating otherwise.

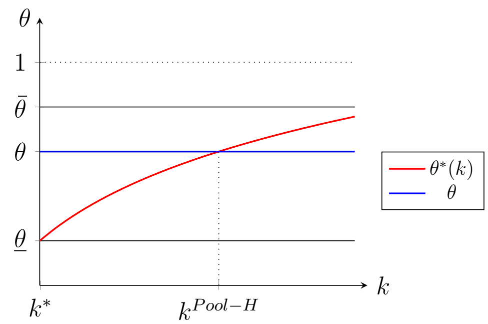


Figure 8: Pooling equilibrium for the interval $[k^*, k^{Pool-H}]$.

The result implies that the commitment trap still exists for high values of the pre-commitment power under private information, as long as the prior belief is not high enough—that is, if $\theta \leq \bar{\theta}$.

To sustain the equilibrium described, we consider the following responder's beliefs regarding the proposer's type after observing the proposer's intended scale x_I:

$$\theta' = P(\alpha = H \mid x_I) = \begin{cases} 1 & \text{if } x_I \leq x^* \\ \theta & \text{if } x_I \in (x^*, x^{B2}) \\ 0 & \text{if } x_I \geq x^{B2}. \end{cases}$$

in which x^* is the intended scale x such that $u_R(x) = \tilde{p}u_R(x(x_I^*, k))$.

If $k > k^{Pool-H}$, the equilibrium is such that the partial-commitment type chooses her optimal intended scale as in the public information case, and the full-commitment type chooses the x such that $u_R(x) = \tilde{p}u_R(x(x_I^*, k))$. For $x_I^L = x_I^*$ to be an equilibrium, it must be that the partial-commitment proposer has no incentives to deviate to an intended scale that assigns probability 1 to the proposer's being the full-commitment type.

For example, if the responder's posterior belief after observing $x_I \in [x^*, x^{B2}]$ is $\theta' = 1$, the partial-commitment type would deviate to $x_I^L = x^R$ and get a higher payoff by pretending to be the full-commitment type. Therefore, it would not be an equilibrium. For the described specification to be an equilibrium, it has to be that $\theta' = \theta$ in that situation.

The beliefs specified above support the equilibrium. The responder's belief is such that $\theta' = \theta$ if $x_I = [x^*, x^{B2}]$, which implies that deviating to x^{B2} renders the partial-commitment proposer worse off. Given that the full-commitment proposer is rational, her best response is to choose x such that $u_R(x) = \tilde{p}u_R(x(x_I^*, k))$.

4.2 The proposer is better off with private information for low k

For the rest of the cases, the partial-commitment proposer is better off under private information only if k is low.

Consider first the case $\tilde{p} > \tilde{p}_M$. Note that if it is common knowledge that the proposer is the full-commitment type, she chooses $x_I = x^R$ as the scale because it maximizes the offer the responder is willing to accept.

Proposition 4 *In the partial-commitment type's payoff-maximizing equilibrium, for $\tilde{p} > \tilde{p}_M$, there is a k^{Pool-H} such that the equilibrium is:*

- *Pooling if $k \leq k^{Pool-H}$ and both proposer types choose $x_I^H(k) = x_I^L(k) = x^R$.*
- *Separating if $k > k^{Pool-H}$ and the intended scales are $x_I^L(k) = x_I^*$ and $x_I^H(k) = x(x_I^*, k)$.*

Both proposer types make the same offer, and the responder accepts the first period.

In this case, the full-commitment type does not start the resolution stage following a rejection. For $\tilde{p} > \tilde{p}_H$, the partial-commitment type also prefers not to do it, even in the absence of private information. However, for $\tilde{p} \in (\tilde{p}_M, \tilde{p}_H]$, the partial-commitment type prefers to start the resolution stage in the absence of private information if k is low. Nevertheless, under private information, the partial-commitment proposer does not start the resolution stage following a rejection, since doing it reveals her type; she prefers to continue negotiating with the same intended scale to mimic the full-commitment type.

We focus on the analysis of $\tilde{p} > \tilde{p}_H$ and relegate to Appendix B.6 the discussion of $\tilde{p} \in (\tilde{p}_M, \tilde{p}_H]$. To sustain the equilibrium described, we consider the following responder's beliefs regarding the proposer's type after observing the proposer's intended scale x_I:

$$\theta' = P(\alpha = H \mid x_I) = \begin{cases} 1 & \text{if } x_I \leq x^* \\ \theta & \text{if } x_I \in (x^*, x^{**}) \\ 0 & \text{if } x_I \geq x^{**}, \end{cases}$$

in which $x^* \leq x^R$ and x^{**} is defined as the $x > x^R$ such that $u_R(x) = u_R(x^*)$. The value of x^* depends on $\tilde{p}$ and k. We relegate to Appendix B.6 the specific values of x^*.

If $k > \bar{k}$, the partial-commitment type can induce the responder to accept the offer $\tilde{p}u_R(x^R)$ by herself by choosing the appropriate x_I^L described in the previous section. If $k \leq \bar{k}$ and $\tilde{p} > \tilde{p}_H$, the partial-commitment proposer does not maximize by herself the offer the responder is willing to accept. Nevertheless, her payoff in a pooling equilibrium is not necessarily better than separating. If the partial-commitment type chooses $x_I^L = x^R$, the offer the responder is willing to accept (assuming the proposer knows she is the partial-commitment type) is $\tilde{p}u_R(x(x^R, k))$, which is lower than $\tilde{p}u_R(x(x_I^*, k))$.

Therefore, the partial-commitment proposer's payoff-maximizing equilibrium is pooling if the cost of choosing a suboptimal intended scale is compensated by the pooling offer the responder is willing to accept:

$$\theta\tilde{p}u_R(x^R) + (1-\theta)\tilde{p}u_R(x(x^R, k)) > \tilde{p}u_R(x(\bar{x}, k)).$$

The value k^{Pool-H} is the k such that the above equation holds with equality. If $k < k^{Pool-H}$, both the partial-commitment and full-commitment types choose x^R as the intended scale.

If $k \in [k^{Pool-H}, \bar{k}]$, the equilibrium is separating and the partial-commitment type chooses $x_I^L = \bar{x}$ and the full-commitment type chooses $x_I^H = x(\bar{x}, k)$. For $x_I^L = \bar{x}$ to be an equilibrium, it must be that the partial-commitment proposer has no incentives to deviate to a threat that puts probability 1 to $\alpha = H$.

For example, if the responder's posterior belief after observing $x_I \in (x(\bar{x}, k), x^{**})$ is $\theta' = 1$, the partial-commitment type would deviate to $x_I^L = x^R$ and get a higher payoff by pretending to be the full-commitment type. Therefore it would not be an equilibrium.

The beliefs specified above support the equilibrium, since the responder's belief is such that $\theta' = \theta$ if $x_I = (x(\bar{x}, k), x^{**})$, which implies that deviating to the neighborhood of x^R renders the partial commitment proposer worse off. The full-commitment type's best response is to choose $x_I^H = x_F^L(\bar{x}, k)$.

Consider now the case $\tilde{p} \leq \tilde{p}_L$. Without private information, the full-commitment type chooses x^{B2} as the intended scale, as described above. In the presence of private information, if k is high and the equilibrium is pooling, the partial-commitment type has incentives to increase the intended scale and continue negotiating following a rejection.

Proposition 5 *In the partial-commitment type's payoff-maximizing equilibrium, for $\tilde{p} < \tilde{p}_L$ there exists $\bar{\theta}$ such that*

- *If $\theta > \bar{\theta}$, the equilibrium is pooling with intended scale $x_I^L = x_I^H = x^{B2}$.*
- *If $\theta \leq \bar{\theta}$, there exists a k^{Pool-L} such that the equilibrium is pooling if $k \leq k^{Pool-H}$ and separating otherwise, in which the intended scales are $x_I^L = x(x_I^*, k)$.*

Both proposer types make the same offer, and the responder accepts in the first period.

For the set of high k such that condition (2) is not satisfied for the partial-commitment type, if she chooses $x_I = x^{B2}$, the equilibrium is pooling if

$$\theta u_R(x^{B2}) + (1-\theta)\Big[pu_R(x(x^{B2}, k)) + (1-p)\delta p u_R(x^R)\Big] \geq u_R(x(x_I^*, k)), \tag{3}$$

and for the set of lower k such that (2) is satisfied for the partial-commitment type, the equilibrium is pooling if

$$\theta u_R(x^{B2}) + (1-\theta) u_R(x(x^{B2}, k)) \geq u_R(x(x_I^*, k)). \tag{4}$$

If $\theta > \bar{\theta}$, the above equations are satisfied for any k. If $\theta \leq \bar{\theta}$, if there is a k^{Pool-L} that satisfies equation (4), that same k^{Pool-L} satisfies equation (5). If there is no k that satisfies equation (4), then k^{Pool-L} is the k that satisfies equation (5). The analysis of the beliefs that sustain the equilibrium is similar to $\tilde{p} > \tilde{p}_H$, and we relegate it to Appendix B.7.

Finally, for $\tilde{p} \in [\tilde{p}_L, \tilde{p}_M]$, if $k < k^*$ the same results as for $\tilde{p} < \tilde{p}_L$ apply, considering only $k < k^*$.

4.3 Effect of private information

If there is no commitment trap, the possible presence of the full-commitment type benefits the partial-commitment proposer if the commitment power is low, since the equilibrium is pooling. In the opposite case, the partial-commitment proposer is better off not pooling with the full-commitment type.

Although the partial-commitment proposer might not reach the highest payoff by herself, she still is better off in a separating equilibrium because the trade-off of pooling is that the proposer must choose a lower intended scale to do so—and therefore, reduces the expected payoff. So, if k is high, the proposer is better off in a separating equilibrium.

In case there is a commitment trap, it still exists under private information for high values of pre-commitment power if the prior belief is not high enough. For lower pre-commitment power, the commitment trap is lessened in the sense that the proposer chooses a lower intended scale, mimicking the full-commitment type, and increases her payoff with respect to the case without private information. Nevertheless, the partial-commitment proposer's payoff is still lower than if she can signal her type and commit to a deterministic threat.

5 Concluding remarks

The proposer's ability to induce the responder to accept a high demand in adversarial bargaining depends on the threat and the credibility of carrying out that threat. This paper studies the effect of a risky environment and scale-down costs on the optimal threat and the credibility of honoring it. Honoring the threat has two dimensions: How much to scale it down in the resolution stage and whether to start the conflict voluntarily.

We show that committing to start the threat is not necessarily the best strategy as long as the proposer can rely on the riskier environment to threaten the responder. We also show that high pre-commitment power can be detrimental to the proposer, since it induces a commitment trap if the probability that the conflict exogenously starts is neither low nor high.

Appendices

A Extensions

A.1 High incentives to start the resolution stage

If $u_R(x^R) > u_P(x^P) > u_R(x^P)$, the same results apply. The only difference is that there is a $\underline{x}$, which is the $x = x(x_I, k)$ that satisfies

$$u_P(x^P) = y(x(x_I, k)),$$

that is, $\underline{x}$ is the scale that makes the proposer indifferent to making the optimal offer $y(x(x_I, k))$, which is accepted by the responder, or making an offer that is rejected and starting the resolution stage with $x_F = x^P$. If $x(x_I, k) < \underline{x}$, the proposer prefers to choose x^P, makes an offer that is rejected for sure, and then starts the resolution stage.

If $u_P(x^P) > u_R(x^R)$, the proposer prefers to choose x^P, makes an offer that is rejected for sure, and then starts the resolution stage.

A.2 The responder's loss is strictly increasing

If the responder's loss is strictly increasing, the maximizer is $x^R = \bar{x}$, as seen in Figure 9:

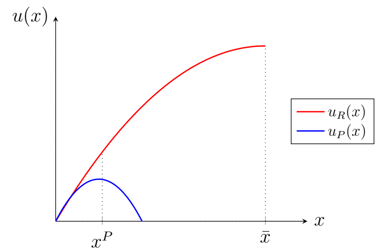


Figure 9: Benefit and loss.

The same qualitative results hold in this context. The difference is that for $\tilde{p} > \tilde{p}_L$, instead of targeting x^R by choosing x_I such that $x(x_I, k) = x^R$, the proposer chooses $x_I = x^R$ for high k. All results remain the same.

A.3 The proposer's payoff is strictly decreasing

If the proposer's benefit is strictly decreasing, such that the proposer's benefit maximizer is $x^P = 0$, as seen in Figure 10, this is equivalent to considering the case $\tilde{p}_H = 0$. That is, the equilibrium is probabilistic brinkmanship for each k. The same results hold.

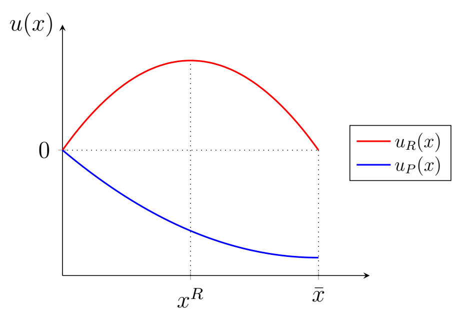


Figure 10: Benefit and loss.

B Proofs

Preliminary definitions and results:

Define $v_P(x, x_I, k) \equiv u_P(x) - kc(x, x_I)$, and $V_P(x_I, k) \equiv u_P(x(x_I, k)) - kc(x(x_I, k), x_I)$. Also, define $x_I(k) \equiv \arg\max_{x_I \in [0,\bar{x}]} v_P(x(x_I, k), x_I, k)$ as the optimal intended scale given k, and $x(k) \equiv x(x_I(k), k)$ as the optimal realized scale given the proposer chooses the optimal intended scale $x_I(k)$.

Lemma B1 *For any x_I, $v_P(x, x_I, k)$ is strictly convex and differentiable*

Proof. As $u_P(x)$ is convex and differentiable, and $-kc(x, x_I)$ is strictly convex and differentiable. Note that if $x \geq x_I$, then $v(x, x_I, k) = u_P(x)$, and if $x < x_I$, then $v(x, x_I, k) < u_P(x)$ for all $k > 0$, because $v(x, x_I, k) < v(x, x_I, k) + kc(x, x_I) = u_P(x)$. It implies $v(x, x_I, k)$ is a convex function always bounded by $u_P(x)$. ■

Lemma B2 *$V_P(x_I, k)$ is strictly decreasing in x_I and k for $x_I \in [x^P, \bar{x}]$.*

Proof. Using the envelope theorem: $\frac{\partial V_P(x_I,k)}{\partial x_I} = -k\frac{\partial c(x(x_I,k),x_I)}{\partial x_I} < 0$ because $\frac{\partial c(x(x_I,k),x_I)}{\partial x_I} > 0$ by assumptions on $c(x(x_I, k), x_I)$. $\frac{\partial V_P(x_I,k)}{\partial K} = -kc(x(x_I, k), x_I) < 0$. ■

B.1 Proof of Lemma 1

For x_I: Take $x'_I > x_I$. Suppose $x(x'_I, k) \leq x(x_I, k)$, then $c(x(x'_I, k), x'_I) > c(x(x'_I, k), x_I)$, and given that $u_P(x(x'_I, k)) - kc(x(x'_I, k), x'_I)$ is optimal, then $V(x_I, k)$ is strictly higher with $x(x'_I, k)$ instead of $x(x_I, k)$, because is the same $u_P(x)$ but lower $c(x, x_I)$. Therefore $x(x_I, k)$ is not the maximizer. Contradiction.

For k: Take $k' > k$. Suppose $x(x_I, k') \leq x(x_I, k)$, then $c(x(x_I, k'), x_I) > c(x(x_I, k), x_I)$, and given that $u_P(x(x_I, k')) - k'c(x(x_I, k'), x_I)$ is optimal, then $V(x_I, k)$ is strictly higher with $x(x_I, k')$ instead of $x(x_I, k)$, because is the same $u_P(x)$ but lower $c(x, x_I)$ that is scaled lower with $k < k'$. Therefore $x(x_I, k)$ is not the maximizer. Contradiction.

B.2 Proof of Proposition 1

For $\tilde{p} > \tilde{p}_H$, $\delta\tilde{p}u_R(x) > u_P(x)$ for all $x \in [x^P, \bar{x}]$. Therefore, $V_P(x_I, k) < \delta\tilde{p}u_R(x(k))$ for all k. The proposer does not start the resolution stage following a rejection. The offer the proposer makes is equal to the responder's continuation loss $y = \tilde{p}u_R(x(k))$. For each $k \geq \bar{k}$, the offer $u_R(x(k))$ is maximized by choosing x_I such that $x(x_I, k) = x^R$, that is, the optimal offer is $y = \tilde{p}u_R(x^R)$ which is accepted by the responder. For $k \in (0, \bar{k})$, the offer is maximized by $x_I = \bar{x}$ and the optimal offer is $y = \tilde{p}u_R(x(\bar{x}, k))$, which is accepted by the responder.

Off the path of equilibrium, if the responder rejects the offer, the proposer does not modify x_I because a probabilistic threat is the only feasible option. For that threat, the proposer is choosing the optimal scale.

For $\tilde{p} < \tilde{p}_L$, then, for any k there is x_I such that the threat is deterministic. For each $k \geq \bar{k}$, there is a $x^*_I(k)$ such that $u_P(x(x_I, k)) - kc(x(x_I, k), x_I) = \delta\tilde{p}u_R(x^R)$, for any $k \geq \bar{k}$, the value $x_I \in [x^P, x^*_I(k)]$ induces a deterministic threat. Condition (1) is satisfies because for any x_I, $u_P(x(x_I, k)) - kc(x(x_I, k), x_I) \geq \delta\tilde{p}u_R(x^R) \geq \delta\tilde{p}u_R(x(x_I, k))$. Condition (2) is satisfied because $u_P(x(x_I, k)) - kc(x(x_I, k), x_I) \geq \delta\tilde{p}u_R(x^R)$. In this case, to satisfy condition

(2) is enough to check that the proposer does not increase the scale to an intended scale $x_F = x^R$.

For each $k \in [\underline{k}, \bar{k}]$, there is a $x_I^{**}(k)$ such that $u_P(x(x_I, k)) - kc(x(x_I, k), x_I) = \delta\tilde{p}u_R(x(\bar{x}, k))$, for any $k \in [\underline{k}, \bar{k}]$, the value $x_I \in [x^P, x_I^{**}(k)]$ induces a deterministic threat. Condition (1) is satisfies because for any x_I, $u_P(x(x_I, k)) - kc(x(x_I, k), x_I) \geq \delta\tilde{p}u_R(x(\bar{x}, k)) \geq \delta\tilde{p}u_R(x(x_I, k))$. Condition (2) is satisfied because $u_P(x(x_I, k)) - kc(x(x_I, k), x_I) \geq \delta\tilde{p}u_R(x(\bar{x}, k))$. Note that in this case, to satisfy condition (2) is enough to check that the proposer does not increase the scale to $x_I = \bar{x}$. The only option is a deterministic threat for each $k < \underline{k}]$.

The proposer prefers a deterministic threat; even the lowest offer $u_R(x^P)$ that the proposer induces the responder to accept is higher or equal to the highest probabilistic threat offer $\tilde{p}u_R(x^R)$. The highest offer the proposer induces is with the highest feasible realized threat $x_F \leq x^R$. Therefore, the proposer chooses an intended scale $x_I = x_I^*(k)$ if $k \geq \bar{k}$, $x_I = x_I^{**}(k)$ if $k \in [\underline{k}, \bar{k}]$, and $x_I = \bar{x}$ if $k < \underline{k}$.

For $\tilde{p} \in [\tilde{p}_L, \tilde{p}_M]$, if $k \leq \underline{k}$, the only option is a deterministic threat and if $k \geq \tilde{k}$ the only option is a deterministic threat. The equilibrium is as described above in the previous cases. If $k \in (\underline{k}, \tilde{k})$, the threat type depends on the intended scale. For any $k \in (\underline{k}, \tilde{k})$, the proposer maximizes the offer that the responder is willing to accept under probabilistic threat by choosing $x_I = \bar{x}$, and call x_{DT} to the optimal x_I that the proposer chooses if she wants to induce a deterministic threat. The value x_{DT} is given by the $x_I(k)$ such that $u_P(x(x_I, k)) - kc(x(x_I, k), x_I) = \delta\tilde{p}u_R(x(\bar{x}, k))$. Call k^* to the value k such that $u_R(x(x_{DT}, k)) = \tilde{p}u_R(x(\bar{x}, k))$. Therefore, if $k \leq k^*$, the proposer chooses a deterministic threat choosing $x_I = x_{DT}$ is $k \in (\underline{k}, k^*]$, and choosing $x_I = \bar{x}$ if $k \leq \underline{k}$. If $k > k^*$, the proposer chooses a probabilistic threat ans chooses $x_I = \bar{x}$ if $k \in (\underline{k}, \bar{k}]$, and chooses x_I such that $x(x_I, k) = x^R$ if $k \leq \bar{k}$.

B.3 Proof of Lemma 2

If $\tilde{p} \leq \tilde{p}_H$, suppose the proposer chooses $x_I = \bar{k}$ and call $\underline{k}$ to the k is such that $u_P(x(\bar{x}, k)) - kc(x(\bar{x}, k), \bar{x}) = \delta\tilde{p}u_R((x(\bar{x}, k))$. The proposer is indifferent between a deterministic and a probabilistic threat. For any $k \leq \underline{k}$ and any x_I, conditions (1) because $u_P(x(x_I, k)) - kc(x(x_I, k), x_I) \geq u_P(x(\bar{x}, k)) - kc(x(\bar{x}, k), \bar{x}) = \delta\tilde{p}u_R((x(\bar{x}, k)) \geq \delta\tilde{p}u_R((x(x_I, k))$. Condition (2) not always satisfies for any x_I, but the best deviation is to choose $x_I = \bar{x}$ that induces a deterministic threat. Therefore, if expecting to increase the scale following a rejection, the proposer chooses $x_I = \bar{x}$ at $t = 1$ and the equilibrium features a deterministic threat.

B.4 Proof of Lemma 3

If $\tilde{p} \geq \tilde{p}_L$, call $\tilde{k}$ to the k such that $u_P(x^P) = \delta\tilde{p}u_R((x(\bar{x}, k))$. For any $k \geq \underline{k}$ the proposer does not satisfied (2) because $u_P(x^P) \leq \delta\tilde{p}u_R(x(\bar{x}, k))$ if $k \in [\tilde{k}, \bar{k})$, and $u_P(x^P) \leq \delta\tilde{p}u_R(x^R)$ if $k \geq \bar{k}$. Therefore, for any $x_I > x^P$, the proposer does not satisfy condition (2) if she wants to induce a deterministic threat.

B.5 Proof of Proposition 2

If $\tilde{p} \in [\tilde{p}_L, \tilde{p}_M]$, then $u_R(x^P) \leq \tilde{p}u_R(x^R)$, even the lowest offer the proposer induces the responder to accept under deterministic threat is higher or equal to the highest offer under probabilistic threat. Therefore, $k^* = \tilde{k}$ because the proposer chooses a x_I to induce a deterministic threat.

B.6 Proof of Proposition 3

For $k \geq k^*$. Consider $\theta^{k^{Pool-H}}$ the θ such that:

$$\theta u_R(x^{B2}) + (1-\theta)\Big[pu_R(x(x^{B2}, k)) + (1-p)\delta\tilde{p}u_R(x(x_I^*, k))\Big] = \tilde{p}u_R(x(x_I^*, k)),$$

in which x_I^* is the optimal x_I for the partial commitment proposer in the absence of private information.

Responder beliefs: In this case, x^* is the x such that $u_(x) = \tilde{p}u_R(x(x_I^L(k), k))$ and $x^{**} = x^{B2}$.

If $\theta \geq \theta^{k^{Pool-H}}$, the equilibrium is pooling, and both proposer types choose x^{B2}. The responder's continuation value of not accepting the offer is $\theta u_R(x^{B2}) + (1-\theta)\Big[pu_R(x(x^{B2}, k)) + (1-p)\delta\tilde{p}u_R(x(x_I^*, k))\Big]$. Therefore, optimal offer of both types is $y = \theta u_R(x^{B2}) + (1-\theta)\Big[pu_R(x(x^{B2}, k)) + (1-p)\delta\tilde{p}u_R(x(x_I^*, k))\Big]$. The proposer does not deviate to a different offer y because y is the highest offer the responder is willing to accept.

In this case, following a rejection, the full-commitment proposer starts the resolution stage. However, the partial commitment proposer does not do it because $pv_P(x^{B2}, k) + (1-p)\delta\tilde{p}u_R(x(x_I^*, k)) > v_P(x^{B2}, k)$. The offer includes that possibility.

If any player deviates from $x_I = x^{B2}$ and chooses $x' \in [x^*, x^{**})$, then $\theta' = \theta$, and the offer that the responder is willing to accept is $\theta u_R(x') + (1-\theta)\Big[pu_R(x(x', k)) + (1-p)\delta\tilde{p}u_R(x(x_I^*, k))\Big] < \theta u_R(x^{B2}) + (1-\theta)\Big[pu_R(x(x^{B2}, k)) + (1-p)\delta\tilde{p}u_R(x(x_I^*, k))\Big]$. If $x' > x^{B2}$, the responder belief would be $\theta' = 0$, and the offer he is willing to accept is $\tilde{p}u_R(x(x', k)) < \theta u_R(x^{B2}) + (1-\theta)\Big[pu_R(x(x^{B2}, k)) + (1-p)\delta\tilde{p}u_R(x(x_I^*, k))\Big]$.

If $\theta < \theta^{k^{Pool-H}}$, the opposite case holds. The equilibrium is separating, in which the full-commitment type chooses x_I^H such that $u_R(x_I^H) = \tilde{p}u_R(x(x_I^L(k), k))$, and the partial commitment type chooses $x_I^L(k)$.

If any player deviates from the equilibrium intended scale and chooses $x' \in (x^*, x^{**})$, the responder's belief would be $\theta' = \theta$ and the offer he is willing to accept is $\theta u_R(x^{B2}) + (1 - \theta)\Big[pu_R(x(x^{B2}, k)) + (1-p)\delta\tilde{p}u_R(x(x_I^*, k))\Big] < \tilde{p}u_R(x(x_I^*, k))$. If $x' > x^{**}$, then $\theta' = 0$, and then the responder is only willing to accept $\tilde{p}u_R(x(x', k)) < \tilde{p}u_R(x(x_I^L(k), k))$. If $x' < x^*$, then $\theta' = 1$ and the offer that the responder is willing to accept is $u_R(x') < \tilde{p}u_R(x(x_I^L(k), k))$. Therefore, it is not optimal for any type to choose a different x_I.

Consider the bounds $\underline{\theta}$:

$$\underline{\theta} \equiv \frac{u_R(x^P) - \Big[pu_R(x(x^{B2}, k)) + (1-p)u_P(x^P)\Big]}{u_R(x^{B2}) - \Big[pu_R(x(x^{B2}, k)) + (1-p)u_P(x^P)\Big]},$$

and $\bar{\theta}$:

$$\bar{\theta} \equiv \frac{p}{1-p} \frac{\Big[u_R(x^R) - u_R(x^{B2})\Big]}{\Big[u_R(x^{B2}) - \delta\tilde{p}u_R(x^R)\Big]}.$$

For each $\theta \in [\underline{\theta}, \bar{\theta}]$, there is a $k^{Pool-H}(\theta)$ such that $\theta > \theta^{k^{Pool-H}}$ if $k < k^{Pool-H}(\theta)$, and $\theta < \theta^{k^{Pool-H}}$ otherwise. Note that $\theta^{k^{Pool-H}}(k)$ is increasing in $k > k^*$ and bounded below by $\underline{\theta}$ and above by $\bar{\theta}$. Define $k^{Pool-H}(\theta)$ as the inverse of $\theta^{k^{Pool-H}}(k)$, which is $k^{Pool-H}(\underline{\theta}) = k^*$ and $k^{Pool-H}(\bar{\theta}) \to \infty$.

B.7 Proof of Proposition 4

For $\tilde{p} \geq \tilde{p}_H$, define k^{Pool-H} as the value k such that:

$$\theta u_R(x^R) + (1-\theta)u_R(x(x^R, k)) = u_R(x(\bar{x}, k)).$$

The equilibrium intended scale for both proposer types are:

$$x_I^L(k) = \begin{cases} x^R & \text{if } k \leq k^{Pool-H} \\ \bar{x} & \text{if } k \in (k^{Pool-H}, \bar{k}] \\ x_I^L(k, \tilde{p}) & \text{if } k > \bar{k} \end{cases} \quad \text{and} \quad x_I^H(k) = \begin{cases} x^R & \text{if } k \leq k^{Pool-H} \\ x_I^H(k, \tilde{p}) & \text{if } k \in (k^{Pool-H}, \bar{k}] \\ x^R & \text{if } k > \bar{k} \end{cases}$$

The value x^* of the responder's beliefs in equation (4) that sustain the equilibrium is defined as the $x \leq x^R$ such that:

$$x^* = \begin{cases} x \text{ such that } u_R(x) = \theta u_R(x^R) + (1-\theta)u_R(x(x^R, k)) & \text{if } k < k^{Pool-H} \\ x_F^L(\bar{x}, k) & \text{if } k \in [k^{Pool-H}, \bar{k}] \\ x^R & \text{if } k > \bar{k} \end{cases}$$

and x^{**} is defined as the $x > x^R$ such that $u_R(x) = u_R(x^*)$. Note that x^* and x^{**} are increasing and decreasing in k respectively, $x^R \in (x^*, x^{**}]$, and that $x^* = x^{**} = x^R$.

For $k \geq \bar{k}$: full-commitment type chooses x^R and the partial-commitment type chooses $x_I(k)$ such that $x(x_I, k) = x^R$, and both types offer $y = \tilde{p}u_R(x^R)$. It is the highest offer that the responder is willing to accept. Therefore, there is no profitable deviation for the proposer. The responder knows the proposer's type after observing x_I and anticipates that by rejecting the offer, his continuation value is $\tilde{p}u_R(x^R)$.

After choosing x_I such that at the resolution stage $x(x_I, k) = x^R$, no other offer is an equilibrium because the responder accepts any offer lower or equal to $\tilde{p}u_R(x^R)$, and reject any offer higher than it. Suppose there is another equilibrium in which the proposer chooses a different initial position than before and reveals information about her type. In that case, at least one proposer's type cannot make an offer equal to $y = \tilde{p}u_R(x^R)$ because it will be rejected.

Suppose the initial position does not reveal the proposer's type. In that case, the proposer makes an offer equal to the responder's continuation value evaluated at a different x, which is lower than $\tilde{p}u_R(x^R)$.

For $k \in [0, \bar{k})$. Consider

$$\theta^{k^{Pool-H}} = \frac{u_R(x(\bar{x}, k)) - u_R(x(x^R, k))}{u_R(x^R) - u_R(x(x^R, k))}.$$

If $\theta < \theta^{k^{Pool-H}}$, the equilibrium is separating, and the full-commitment type chooses $x_I^H = x(\bar{x}, k)$ and the partial commitment type chooses $x_I^L = \bar{x}$.

If any player deviates from the equilibrium intended scale and chooses $x' \in (x^*, x^{**})$, the responder's belief would be $\theta' = \theta$ and the offer he is willing to accept is $\tilde{p}[\theta u_R(x') + (1 - \theta)u_R(x(x', k))] < \tilde{p}u_R(x(\bar{x}, k))$. If $x' > x^{**}$, then $\theta' = 0$, and then the responder is only willing to accept a low offer $\tilde{p}u_R(x(x', k)) < \tilde{p}u_R(x(\bar{x}, k))$. If $x' < x_I^H(k, \tilde{p})$, then $\theta' = 1$ and the offer that the responder is willing to accept is $\tilde{p}u_R(x') < \tilde{p}u_R(x_I^H(k, \tilde{p}))$. Therefore, it is not optimal for any type to choose a different x_I.

If $\theta \geq \theta^{k^{Pool-H}}$, the equilibrium is pooling, and both proposer types choose x^R. The responder's continuation value of not accepting the offer is $\tilde{p}[\theta u_R(x^R) + (1-\theta)u_R(x(x^R, k))]$. Therefore, optimal offer of both types is $y = \tilde{p}[\theta u_R(x^R) + (1-\theta)u_R(x(x^R, k))]$. The proposer does not deviate to a different offer y because y is the highest offer the responder is willing to accept.

If any player deviates from $x_I = x^R$ and chooses $x' \in (x^*, x^{**})$, then $\theta' = \theta$ and the offer that the responder is willing to accept is $\tilde{p}[\theta u_R(x') + (1-\theta)u_R(x(x', k))] < \tilde{p}[\theta u_R(x^R) + (1-\theta)u_R(x(x^R, k))]$. If $x' < x^*$, the responder belief would be $\theta' = 1$ and the offer he is willing to accept is $\tilde{p}u_R(x') < \tilde{p}[\theta u_R(x^R) + (1-\theta)u_R(x(x^R, k))]$. And if $x' > x^{**}$, the responder belief would be $\theta' = 0$, and the offer he is willing to accept is $\tilde{p}u_R(x(x', k)) \leq \tilde{p}u_R(x(\bar{k}, k)) < \tilde{p}[\theta u_R(x^R) + (1-\theta)u_R(x(x^R, k))]$. Therefore, it is not optimal for any type to choose a different x_I.

For each θ, there is a k^{Pool-H} such that $\theta > \theta^{k^{Pool-H}}$ if $k < k^{Pool-H}$, and $\theta < \theta^{k^{Pool-H}}$ otherwise. The value k^{Pool-H} is increasing in θ for $\theta \in [0,1]$, with $k^{Pool-H}(0) = 0$ and $k^{Pool-H}(1) = \bar{k}$.

Note that $\theta^{k^{Pool-H}}$ belongs to $(0,1)$ as $\lim_{k \to 0} \theta^* = 0$ because $\lim_{k \to 0} u_R(x(\bar{x}, k)) = u_R(x^P)$ and $\lim_{k \to 0} u_R(x(x^R, k)) = u_R(x^P)$. Also $\lim_{k \to \bar{k}} \theta^* = 1$ because $\lim_{k \to \bar{k}} u_R(x(\bar{x}, k)) = u_R(x^R)$. Also, $\theta^{k^{Pool-H}}(k)$ is monotone in $[0, \bar{k}]$, as the cutoff $k(\theta^{k^{Pool-H}})$ is unique for θ, otherwise there exists a θ such that there is more than a cutoff $k(\theta^{k^{Pool-H}})$. It implies that in equilibrium there exists $k \in [k(\theta^{*1}), k(\theta^{*2})]$ such that the equilibrium is separating and pooling if $k < k(\theta^{*1})$ or $k > k(\theta^{*1})$. That situation is not possible, because $u_R(x(\bar{x}), k)$ is monotone increasing in k, with $u_R(x(\bar{x}), 0) = u_R(x^P)$ and $u_R(x(\bar{x}), \bar{k}) = u_R(x^R)$, and $\theta u_R(x^R) + (1-\theta)u_R(x(x^R, k)$ is monotonically increasing in $[\theta u_R(x^R) + (1-\theta)u_R(x(x^R, 0), \theta u_R(x^R) + (1-\theta)u_R(x(x^R, \bar{k})]$. Therefore there is only one k such that $\theta u_R(x^R) + (1-\theta)u_R(x(x^R, k) \geq u_R(x(\bar{x}), k)$. Therefore, taking the inverse of $\theta^{k^{Pool-H}}(k)$, we have that $k^*(\theta)$ is increasing in θ.

The same proof applies for $\tilde{p} \in (\tilde{p}_M, \tilde{p}_H)$. Note that for the partial-commitment proposer, if $x(x^R, k)$ is such that condition (1) is satisfied, the partial-commitment proposer still prefers not to start the resolution stage, as not revealing information is optimal because:

$$v_P(x_I, k) < \delta\big[\theta u_R(x^R) + (1-\theta)u_R(x(x^R, k))\big].$$

Note that $\delta u_R(x^R)$ and $\delta u_R(x(x^R, k))$ are both larger than $v_P(x_I, k)$ for $\tilde{p} > \tilde{p}_M$ and $\delta \geq \bar{\delta}$.

B.8 Proof of Proposition 5

The responder's equilibrium beliefs regarding the proposer type after observing the chosen intended scale are:

$$\theta' = P(\alpha = H \mid x_I) = \begin{cases} 1 & \text{if } x_I \leq x^* \\ \theta & \text{if } x_I \in (x^*, x^{**}) \\ 0 & \text{if } x_I \geq x^{**}. \end{cases}$$

Consider k^D as the k such that $v_P(x^{B2}, k) = \delta \tilde{p} u_R(x(x_I^*, k))$, in which x_I^* is the optimal intended scale the partial-commitment proposer chooses in the absence of private information. Note $k^D > \underline{k}$.

For $k > k^D$. Consider

$$\theta^{k^{Pool-L-1}} = \frac{u_R(x(x_I(k), k)) - \left[p u_R(x(x^{B2}, k)) + (1-p)\delta \tilde{p} u_R(x(x_I^*, k))\right]}{u_R(x^{B2}) - \left[p u_R(x(x^{B2}, k)) + (1-p)\delta \tilde{p} u_R(x(x_I^*, k))\right]}.$$

Responder beliefs: In this case the value x^* is defined as the highest value between the optimal election of the partial commitment type if the information were public $x(x_I^L(x), k)$ and the x such that $u_R(x) = \theta u_R(x^{B2}) + (1-\theta)\left[p u_R(x(x^{B2}, k)) + (1-p)\delta \tilde{p} u_R(x(x_I^*, k))\right]$. And x^{**} is defined as the $x > x^R$ such that $u_R(x) = u_R(x^*)$.

If $\theta \geq \theta^{k^{Pool-L-1}}$, the equilibrium is pooling, and both proposer types choose $k^{Pool-L-1}$. The responder's continuation value of not accepting the offer is $\theta u_R(x^{B2}) + (1-\theta)\left[p u_R(x(x^{B2}, k)) + (1-p)\delta \tilde{p} u_R(x(x_I^*, k))\right]$. Therefore, optimal offer of both types is $y = \theta u_R(x^{B2}) + (1-\theta)\left[p u_R(x(x^{B2}, k)) + (1-p)\delta \tilde{p} u_R(x(x_I^*, k))\right]$. The proposer does not deviate to a different offer y because y is the highest offer the responder is willing to accept.

If any player deviates from $x_I = x^{B2}$ and chooses $x' \in (x^*, x^{**})$, then $\theta' = \theta$, and the offer that the responder is willing to accept is $\theta u_R(x') + (1-\theta)\left[p u_R(x(x', k)) + (1-p)\delta \tilde{p} u_R(x(x_I^*, k))\right] < \theta u_R(x^{B2}) + (1-\theta)\left[p u_R(x(x^{B2}, k)) + (1-p)\delta \tilde{p} u_R(x(x_I^*, k))\right]$. If $x' < x^*$, the responder belief would be $\theta' = 1$ and the offer he is willing to accept is $u_R(x') < \theta u_R(x^{B2}) + (1-\theta)\left[p u_R(x(x^{B2}, k)) + (1-p)\delta \tilde{p} u_R(x(x_I^*, k))\right]$. And if $x' > x^{**}$ the responder belief would be $\theta' = 0$, and the offer he is willing to accept is $\tilde{p} u_R(x(x', k)) \leq \theta u_R(x^{B2}) + (1-\theta)\left[p u_R(x(x^{B2}, k)) + (1-p)\delta \tilde{p} u_R(x(x_I^*, k))\right]$. Therefore, it is not optimal for any type to choose a different x_I.

If $\theta < \theta^{k^{Pool-L-1}}$, the opposite case holds. The equilibrium is separating, in which the full-commitment type chooses $x_I^H = x(x_I^L(k), k)$, and the partial commitment type chooses $x_I^L(k)$.

If any player deviates from the equilibrium intended scale and chooses $x' \in (x^*, x^{**})$, the responder's belief would be $\theta' = \theta$ and the offer he is willing to accept is $\theta u_R(x') + (1-\theta)\big[pu_R(x(x',k))+(1-p)\delta\tilde{p}u_R(x(x_I^*,k))\big] < u_R(x(x_I^L(k),k))$. If $x' > x^{**}$, then $\theta' = 0$, and then the responder is only willing to accept a low offer $\tilde{p}u_R(x(x',k)) < u_R(x(x_I^L(k),k))$. If $x' < x^*$, then $\theta' = 1$ and the offer that the responder is willing to accept is $u_R(x') < u_R(x(x_I^L(k),k))$. Therefore, it is not optimal for any type to choose a different x_I.

For each θ, there is a $k^{Pool-L-1}(\theta)$ such that $\theta > \theta^{k^{Pool-L-1}}$ if $k < k^{Pool-L-1}(\theta)$, and $\theta < \theta^{k^{Pool-L-1}}$ otherwise. Note that $\theta^{k^{Pool-L-1}}(k)$ is increasing in $k > \bar{k}$ and bounded below by $\underline{\theta}^{k^{Pool-L-1}}$ and above by $\bar{\theta}^{k^{Pool-L-1}}$:

$$\underline{\theta}^{k^{Pool-L-1}} = \frac{u_R(x(x^{B2},k^D)) - u_R(x(x^{B2},k^D))}{u_R(x^{B2}) - u_R(x(x^{B2},k^D))}, \quad \text{and}$$

$$\bar{\theta}^{k^{Pool-L-1}} = \frac{u_R(x^B) - \big[pu_R(x^{B2}) + (1-p)\delta\tilde{p}u_R(x^R)\big]}{u_R(x^{B2}) - \big[pu_R(x^{B2}) + (1-p)\delta\tilde{p}u_R(x^R)\big]}.$$

Define $k^{Pool-L-1}(\theta)$ as the inverse of $\theta^{k^{Pool-L-1}}(k)$, which is $k^{Pool-L-1}(\theta) = \bar{k}$ for $\theta \leq \underline{\theta}^{k^{Pool-L-1}}$, and $k^{Pool-L-1}(1) = \tilde{k}$.

Note that $\theta^{k^{Pool-H}}(k)$ is increasing in $k > k^D$ and bounded below by $\underline{\theta}$ and above by $\bar{\theta}$. For $\theta \in [\underline{\theta}, \bar{\theta}]$, define $k^{Pool-L-1}(\theta)$ as the inverse of $\theta^{k^{Pool-L-1}}(k)$. Note that the equilibrium is pooling for any k if $\theta < \underline{\theta}^{k^{Pool-L-1}}$, the equilibrium is separating for any k if $\theta > \bar{\theta}^{k^{Pool-L-1}}$, and depend on k otherwise.

For $k \in [0, k^D)$. Consider

$$\theta^{k^{Pool-L-2}} = \frac{u_R(x(x_I^*,k)) - u_R(x(x^{B2},k))}{u_R(x^{B2}) - u_R(x(x^{B2},k))}.$$

Responder beliefs: In this case, the value x^* is defined as the highest value between the optimal election of the partial commitment type if the information were public $x(\bar{x},k)$ and the x such that $u_R(x) = \theta u_R(x^{B2}) + (1-\theta)u_R(x(x^{B2},k))$. And x^{**} is defined as the $x > x^R$ such that $u_R(x) = u_R(x^*)$.

If $\theta < \theta^{k^{Pool-L-2}}$, the equilibrium is separating, and the full-commitment type chooses $x_I^H = x(x_I^*,k)$ and the partial commitment type chooses $x_I^L = x_I^*$.

If any player deviates from the equilibrium initial intended scale and chooses $x' \in (x^*, x^{**})$, the responder's belief would be $\theta' = \theta$ and the offer he is willing to accept is $\theta u_R(x') + (1-\theta)u_R(x(x',k)) < u_R(x(x_I^*,k))$. If $x' > x^{**}$, then $\theta' = 0$, and then the responder is only willing to accept a low offer $u_R(x(x',k)) < u_R(x(x_I^*,k))$. If $x' < x_I^H(k,\tilde{p})$, then $\theta' = 1$ and the offer that the responder is willing to accept is $u_R(x') < u_R(x_I^H(k,\tilde{p}))$. Therefore, it is not optimal for any type to choose a different x_I.

If $\theta \geq \theta^{k^{Pool-L-2}}$, the equilibrium is pooling, and both proposer types choose x^{B2}. The responder's continuation value of not accepting the offer is $\theta u_R(x^{B2}) + (1-\theta)u_R(x(x^{B2},k))$. Therefore, optimal offer of both types is $y = \theta u_R(x^{B2}) + (1-\theta)u_R(x(x^{B2},k))$. The proposer does not deviate to a different offer y because y is the highest offer the responder is willing to accept.

If any player deviates from $x_I = x^{B2}$ and chooses $x' \in (x^*, x^{**})$, then $\theta' = \theta$, and the offer that the responder is willing to accept is $\theta u_R(x') + (1-\theta)u_R(x(x',k)) < \theta u_R(x^{B2}) + (1-\theta)u_R(x(x^{B2},k))$. If $x' < x^*$, the responder belief would be $\theta' = 1$ and the offer he is willing to accept is $u_R(x') < \theta u_R(x^{B2}) + (1-\theta)u_R(x(x^{B2},k))$. And if $x' > x^{**}$, the responder belief would be $\theta' = 0$, and the offer he is willing to accept is $u_R(x(x',k)) \leq u_R(x(x_I^*,k)) < \theta u_R(x^{B2}) + (1-\theta)u_R(x(x^{B2},k))$. Therefore, it is not optimal for any type to choose a different x_I.

For each θ, there is a $k^{Pool-L-2}$ such that $\theta > \theta^{k^{Pool-L-2}}$ if $k < k^{Pool-L-2}$, and $\theta < \theta^{k^{Pool-L-2}}$ otherwise. Note that $\theta^{k^{Pool-L-2}}(k)$ is increasing in $k \in [0, k^D]$ and bounded below by 0 and above by $\bar{\theta}^{k^{Pool-L-2}}$, as the limit when $k \to 0$ is 0, and

$$\bar{\theta}^{k^{Pool-L-2}} \equiv \frac{u_R(x(x^{B2},k^D)) - u_R(x(x^{B2},k^D))}{u_R(x^{B2}) - u_R(x(x^{B2},k^D))}.$$

Define $k^{Pool-L-2}(\theta)$ as the inverse of $\theta^{k^{Pool-L-2}}(k)$.

Note that $\bar{\theta}^{k^{Pool-L-2}} = \underline{\theta}^{k^{Pool-L-1}}$. Then, define $\theta^*(k)$ as

$$\theta^*(k) = \begin{cases} \frac{u_R(x(x_I^*,k)) - u_R(x(x^{B2},k))}{u_R(x^{B2}) - u_R(x(x^{B2},k))} & \text{if } k \in [0,\bar{k}) \\ \frac{u_R(x(x_I(k),k)) - \left[p u_R(x(x^{B2},k)) + (1-p)\delta\tilde{p} u_R(x(x_I^*,k))\right]}{u_R(x^{B2}) - \left[p u_R(x(x^{B2},k)) + (1-p)\delta\tilde{p} u_R(x(x_I^*,k))\right]} & \text{if } k \geq \bar{k}. \end{cases}$$

The function $\theta^*(k)$ is bounded above by:

$$\bar{\theta} = \frac{u_R(x^B) - \left[p u_R(x^{B2}) + (1-p)\delta\tilde{p} u_R(x^R)\right]}{u_R(x^{B2}) - \left[p u_R(x^{B2}) + (1-p)\delta\tilde{p} u_R(x^R)\right]},$$

and it is continuous and strictly increasing. Define $k^{Pool-L}(\theta)$ as the inverse of $\theta^*(k)$.

For $\tilde{p} \in [\tilde{p}_L, \tilde{p}_M]$ and $k < k^*$. The same proof as above applies. The only difference in this case $\theta^*(k)$ is bounded above by:

$$\bar{\theta} = \frac{u_R(x_I^*(k^*)) - \left[pu_R(x^{B2}) + (1-p)\delta\tilde{p}u_R(x^R)\right]}{u_R(x^{B2}) - \left[pu_R(x^{B2}) + (1-p)\delta\tilde{p}u_R(x^R)\right]},$$

in which $x_I^*(k^*)$ is given by $u_P(x(x_I, k)) - kc(x(x_I, k), x_I) = \delta\tilde{p}u_R(x(\bar{x}, k))$.

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